



Math 105 History of Mathematics

First Test

February 2015

You may refer to one sheet of notes on this test. Points for each problem are in square brackets. You may do the problems in any order that you like, but please start your answers to each problem on a separate page of the bluebook. Please write or print clearly.

Problem 1. Essay. [30] Select *one* of the three topics A, B, and C.

Please think about these topics and make an outline before you begin writing. You will be graded on how well you present your ideas as well as your ideas themselves. Each essay should be relatively short—one to three written pages. There should be no fluff in your essays. Make your essays well-structured and your points as clearly as you can.

Topic A. Explain the logical structure of the *Elements* (axioms, propositions, proofs). How does this differ from earlier mathematics of Egypt and Babylonia? How can such a logical structure affect the mathematical advances of a civilization?

Topic B. Compare and contrast the arithmetic of the Babylonians with that of the Egyptians. Be sure to mention their numerals, algorithms for the arithmetic operations, and fractions. Illustrate with examples. Don't go into their algebra or geometry for this essay.

Topic C. Aristotle presented four of Zeno's paradoxes: the Dichotomy, the Achilles, the Arrow, and the Stadium. Select one, but only one, of them and write about it. State the paradox as clearly and completely as you can. Explain why it was considered important. Refute the paradox, either as Aristotle did, or as you would from a modern point of view.

Problem 2. [10] Find the greatest common divisor of the two numbers 11484 and 7902 by using the Euclidean algorithm. (Computations are sufficient, but show your work. An explanation is not necessary.)

Problem 3. [10] Multiply the Egyptian fractions $7 \frac{2}{4} \frac{8}{8}$ by $12 \frac{3}{3}$ using the Egyptian multiplication technique. Note that it is necessary to multiply each term of the multiplicand by $\frac{1}{3}$ separately.

Problem 4. [15] Explain the origins of the sexagesimal system (base 60) used in Mesopotamia. (A one-paragraph answer is sufficient.)

Problem 5. [15] On areas of circles.

- a. How did the Egyptians and Babylonians compute the areas of circles?
- b. What is the problem of quadrature of circle that vexed the Greeks so much?

Problem 6. [20; 4 points each part] True/false. For each sentence write the whole word “true” or the whole word “false”. If it’s not clear whether it should be considered true or false, you may explain in a sentence if you prefer.

a. Euclid’s parallel postulate (Postulate 5 in Book I of the *Elements*) stated that lines in the same direction are parallel.

b. The ancient Babylonians knew the Pythagorean theorem (the square on the hypenense of a right triangle is equal to the sum of the squares on the other two sides) over a thousand years before Pythagoras.

c. Each of the propositions in Euclid’s *Elements* includes a proof.

d. A triangular number is the perimeter of an equilateral triangle, for example, 15 is a triangular number since an equilateral triangle of side length 5 has perimeter 15.

e. Whereas Egyptians used common fractions like $\frac{2}{5}$, Babylonians preferred unit fractions like one-third plus one-fifteenth.

Bonus question. Is the dress white and gold, or is it blue and black?

