

Math 121

Calculus II

Spring 2004

Test #1

Name: _____

Section 1 2 3

Instructor: _____

This exam is CLOSED NOTES and CLOSED BOOK. There are NO CALCULATORS allowed. To get full credit you must show all work neatly in the space provided on the test paper.

1. Compute the following integrals:

[9 pts each]

a. $\int 5x^2 - 3 + \sec^2(x) dx$

b. $\int \frac{x^2}{\sqrt{3-x^3}} dx$

c. $\int \sin(t) \cos^4(t) dt$

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d. $\int_0^1 \frac{3t}{(t^2 + 1)^2} dt$

e. $\int_0^\pi x^2 - \pi^2 dx$

[10 pts]

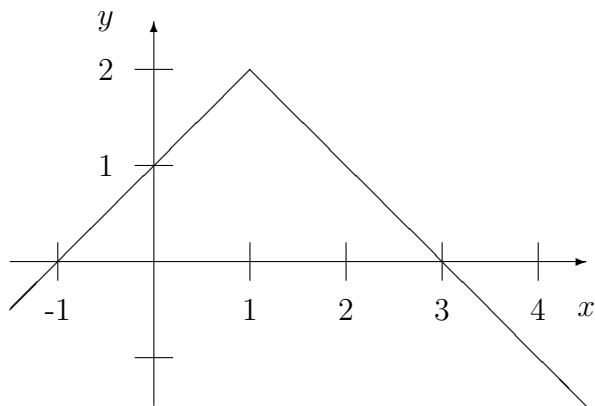
2. Use the substitution $u = 3x$ to transform the integral $\int_2^3 x^3 \sin(3x) \sqrt{x+1} dx$ into an integral with respect to u . [Do not evaluate the integral.]

3. Find the area of the finite region bounded by the graphs of the two [10 pts]
functions $f(x) = 2x - x^2$ and $g(x) = -x$.

[10 pts]

4. Let $f(x) = 2 - |1 - x|$. Let P be the partition

$$P = \left\{ -1, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, 4 \right\}.$$



a. On the graph of $f(x)$ above, draw the rectangles corresponding to $U_f(P)$.

b. If $Q = \left\{ -1, \frac{-1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4 \right\}$, Is $U_f(Q)$ greater than or less than $U_f(P)$? Why?

c. If $R = \{-1, 0, 1, 2, 3, 4\}$, which is the smallest, $U_f(P)$, $U_f(Q)$, $U_f(R)$, or $\int_{-1}^4 f(x) dx$? Why?

5. Suppose that the acceleration of a particle along the x axis is given [10 pts]
 $a(t) = 2t - 10$ for $0 \leq t \leq 5$, and suppose that at $t = 5$ the particle is
at rest at the origin, that is, $x(5) = 0$ and $v(5) = 0$.

What is the position of the particle at $t = 0$?

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[5 pts]

6. Let $f(x)$ be continuous on $[0, \pi]$. Show that $\int_0^\pi 2 + [f(x)]^2 dx > 6$.

[10 pts]

7. Define a function $F(x)$ by $F(x) = \int_0^x \frac{1}{1+t^2} dt$.

a. Compute $F(0)$.

b. Compute $F'(x)$.

c. Show that $F(x)$ is increasing for all x .

d. Determine for which x values $F(x)$ is concave up, and for which it is concave down.

e. Circle the graph of $F(x)$ among the choices below.

