Math 114 Discrete Math Midterm

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Your name: ____________________________

You may use one sheet of prepared notes during this test. Show all your work for credit. Points for each problem are in square brackets.

Problem 1. Translation into symbolic expressions. [12] Let p, q, and r be abbreviations for the following propositions

p: "Grizzly bears have been seen in the area,"
q: "Hiking is safe on the trail," and
r: "Berries are ripe along the trail."

Write these propositions using p, q, and r and logical connectives.

a. Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b. If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been see in the area.

c. Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Problem 2. On truth tables. [10] Use a truth table to show that $(p \to r) \wedge (q \to r)$ is logically equivalent to $(p \vee q) \to r$.

Problem 3. Interpretation of symbolic expressions. [15] Determine the truth value of each of the following statements if the universe of discourse for each variable consists of all real numbers. Simply write "true" or "false" for each; no need to explain why.

__________ **a.** $\forall x\, \exists y\, (x = y^2)$.

__________ **b.** $\exists x\, \exists y\, (x + y \neq y + x)$.

__________ **c.** $\forall x{\neq}0\; \exists y\, (xy = 1)$.

__________ **d.** $\exists y\, \forall x{\neq}0\, (xy = 1)$.

__________ **e.** $\forall x\, \exists y\, (2x + 3y = 7)$.

Problem 4. On rules of inference. [10] Draw relevant conclusions from statements 1, 2, and 3 below, and state what rules of inference you use. (If you don't know the name for a rule of inference, then just state the whole rule symbolically.)

1. "If I play hockey, then I am sore the next day."

2. "I use the whirlpool if I am sore."

3. "I did not use the whirlpool."

Problem 5. On the growth of functions. [10] Which of these following functions are $O(n^3)$? Simply write "yes" or "no" for each; no need to explain why.

__________ **a.** $f(x) = 4x^3 + 5x - 8$.

__________ **b.** $f(x) = x^2 \log x$.

__________ **c.** $f(x) = 2^x$.

Problem 6. On algorithm analysis. [10] Here's an algorithm that will sort an array of integers $a_1, a_2, \ldots, a_n$ of length n into increasing order. First it will be described verbally, then in pseudocode.

Scan down the entire array comparing adjacent pairs of elements, and switch them if they're out of order. Perform this whole array scan $n-1$ times.

In the following pseudocode, comments are placed in braces {}.

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For i = 1 to n − 1 {i is used to count the number of scans}
    For j = 1 to n − 1 {j is used travel down the array}
        If a_j > a_{j+1} then exchange a_j with a_{j+1}
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a. In this algorithm, exactly how many times will adjacent pairs of elements be compared? That is, in the pseudocode, how many times will the comparison $a_j > a_{j+1}$ be executed? (Your answer will be expressed in terms of n. Hint: how many scans? how many comparisons per scan?)

b. What is the time complexity of this sorting algorithm measured in terms of comparisons? That is, what is a big-O estimate for the amount of time that it will take to execute?

Problem 7. On set theory and its notation. [12] Let A, B, and C be the following sets.

$$A = \{1, 3, 5, 7, 9\} \qquad B = \{2, 3, 6, 7, 10\} \qquad C = \{1, 2, 6, 9\}.$$

For each of the following statements, simply write "true" or "false"; no need to explain why.

__________ **a.** The symmetric difference $A \oplus B$ of A and B is $\{1, 2, 5, 6, 9, 10\}$.

__________ **b.** $A \cap B$ is disjoint from C.

__________ **c.** The cardinality of $A \times B$ is 10.

__________ **d.** The cardinality of the power set of C is 16.

Problem 8. On modular arithmetic. [10] Find an integral solution to the congruence

$$5n \equiv 1 \pmod{11}.$$

Problem 9. On the Euclidean algorithm. [10]

a. Use the Euclidean algorithm to determine the greatest common divisor of the two integers 667 and 437. Show your work.

b. Use the results of part a to reduce the fraction 437/667 to lowest terms.