Math 121 | Calculus II | Spring 2005

Test #3

Hour 9AM 10AM 11AM

Name: ___________________

Instructor: _______________

This exam is CLOSED NOTES and CLOSED BOOK. There are NO CALCULATORS allowed. To get full credit you must show all work neatly in the space provided on the test paper.

1. (**24 pts**) Evaluate the following derivatives

a. $\frac{d}{dx}\left(\frac{1}{\ln(x^2+1)}\right)$

b. $f'(0)$, where $f(x) = e^{-x}(3\sin(x) + 2\cos(x))$

c. $f(x) = e^x \arctan(x)$

d. $f'(1/2)$, where $f(x) = \arcsin(\sqrt{x})$

2. (**24 pts**) Evaluate the following integrals.

a. $\displaystyle\int_1^e \frac{1+x}{x^2}\,dx$

b. $\displaystyle\int (s+1)\,\cos(s\,e^s)\,e^s\,ds$

c. $\displaystyle\int \frac{\sin(t)}{5+\cos^2(t)}\,dt$

d. $\displaystyle\int \frac{dx}{x\,\sqrt{1-(\ln(x))^2}}$

3. (**12 pts.**) Let $f(x) = x^3 + e^x$.

a. Calculate $f(0)$ and $f(-1)$.

b. Verify that $f$ is one-to-one on $(-\infty, \infty)$.

c. Calculate $(f^{-1})'(1)$ and $(f^{-1})'(1+e)$.

4. (**8 pts.**) Use the properties of logarithms to rewrite $\ln \dfrac{e^4\, x^5}{\sqrt{x^3+2}}$ as an equivalent expression in terms of sums and differences of logarithms, in simplest terms.

5. (**12 pts.**) a. Find the real numbers $A$ and $k$ such that the function $f(x) = Ae^{kt}$ satisfies $f(1) = 100$ and $f(3) = 10{,}000$.

b. For the function $f(x)$ determined by your answer to (a), give the **exact** numerical values $f(0)$, $f(2)$, and $f(4)$.

6. (**10 pts.**) a. Sketch the region bounded by the lines $x = \pi/4$ and $x = 3\pi/4$ and the graphs $y = \sin(x)$ and $y = \csc(x)$.

b. Find the area of that region.

7. (**10 pts.**) a. Sketch the region bounded by the graphs of $y = x^2$ and $y = 8/(x^2 + 4)$.

b. Find the area of that region.

| Prob | Pts |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| Total | |