Name: ____________________________

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Math 122 Calculus III

Second Test, November 2011

You may use a calculator and a sheet of notes. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

Problem 1. On curves in polar coordinates. [10]

Convert the equation $r = \sin\theta + \sin 2\theta$ of a curve given in polar coordinates to one in rectangular coordinates, x and y.

Problem 2. On areas in polar coordinates. [10]

Write down an integral which gives the area inside the curve $r = \sin\theta + \sin 2\theta$ between the angles $\theta = 0$ and $\theta = \pi/4$. Do not evaluate the integral.

Problem 3. On the concept of convergence of a series. [21; 7 points each part]

a. Complete the definition of convergence. The series $a_1 + a_2 + \cdots + a_n + \cdots$ converges to the sum S if

b. Consider the geometric series $\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \cdots + \left(\frac{4}{5}\right)^n + \cdots$. Evaluate the n partial sum $S_n = \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \cdots + \left(\frac{4}{5}\right)^n$

c. Use what you found in part **b** and the definition of convergence to show that the series in part **b** converges to the sum $S = 4$.

Problem 4. On convergence of series. [40; 10 points each] Determine whether the following series converge or diverge. Name any convergence tests you use and show any relevant computations.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^{3/2}}$

b. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n^2-1}{4n^2+5n+2}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2-1}{4n^4+5n+2}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{20}}{10^n}$

Problem 5. On power series. [20; 10 points each part]

a. Determine the interval of convergence of the power series $\sum_{n=0}^{\infty} nx^n$.

b. Suppose a function f has the following derivatives when evaluated at 0. $f(0) = 0$, $f'(0) = 1$, $f''(0) = 2$, $f'''(0) = 3$, and in general $f^{(n)}(0) = n$. Write down the only power series that represents f;

$$f(x) = \sum_{n=0}^{\infty} a_n x^n =$$

#1.[10]	
#2.[10]	
#3.[21]	
#4.[40]	
#5.[20]	
Total	