Name: _____

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Math 122 Calculus III

Final Exam, December 2011

You may use a calculator and a sheet of notes. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [18] On vector operations. For vectors $\mathbf{a} = (2, -3, 1)$ and $\mathbf{b} = (-3, 1, 9)$ in $\mathbf{R}^3$, compute the following

a. $4\mathbf{a} - 3\mathbf{b} =$

b. $\mathbf{a} \cdot \mathbf{b} =$

c. $\mathbf{a} \times \mathbf{b} =$

d. $\|\mathbf{a}\| =$

e. Find a unit vector $\mathbf{u}$ in the direction of $\mathbf{a}$¿

f. The angle θ between $\mathbf{a}$ and $\mathbf{b}$.

2. [20; 10 points each part] On arclength and surface area. Consider the curve given parametrically by $(x, y) = (2\cos t, 3\sin t)$ for $t \in [0, \pi/2]$. This is part of an ellipse in the first quadrant of the plane.

a. Write down an integral which gives the length of this curve. (Don't evaluate the integral.)

b. This curve is rotated about the x-axis to produce a surface of revolution. Write down an integral which gives the area of this surface. (Don't evaluate the integral.)

3. [16; 8 points each part] On areas and volumes.

a. Determine the area of a parallelogram in $\mathbf{R}^2$ where one side is the vector $\mathbf{v} = (4, -5)$ and another side is $\mathbf{w} = (2, 3)$.

b. Determine the volume of a parallelpiped in $\mathbf{R}^3$ with edges $\mathbf{u} = (1, 8, 2)$, $\mathbf{v} = (3, 0, -5)$, and $\mathbf{w} = (2, -1, 3)$.

4. [18; 6 points each part] On velocity, speed, and acceleration.

Consider a point moving in time whose position at time t is $\mathbf{x} = (t, t^3, 5 - t^2)$.

a. What is the velocity of this particle at time t?

b. What is its speed at time t?

c. What is its acceleration at time t?

5. [12; 4 points each part] On orthogonality of vectors.

Consider the three vectors $\mathbf{u} = (2, 2, 4)$, $\mathbf{v} = (1, 2, 1)$, and $\mathbf{w} = (3, -1, -1)$. Determine which pairs are orthogonal, that is, at right angles.

a. Is $\mathbf{u} \perp \mathbf{v}$?

b. Is $\mathbf{u} \perp \mathbf{w}$?

c. Is $\mathbf{v} \perp \mathbf{w}$?

6. [16] Compute the determinant

$$\begin{vmatrix} 1 & 2 & 0 & -1 \\ 3 & 5 & 2 & 2 \\ 2 & 4 & 1 & 3 \\ 0 & 0 & 2 & 1 \end{vmatrix}$$

#1.[18]	
#2.[20]	
#3.[16]	
#4.[18]	
#5.[12]	
#6.[16]	
Total	