Name: _____

Mailbox number:_____

Math 122 Calculus III

Second Test, November 2012

You may use a calculator and a sheet of notes. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [12] Determine the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

2. [40; 10 points each part] On convergence of series with positive terms. For each series, apply one or more convergence tests to determine whether the series converges. Be sure to mention which test you use.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n!}$

c. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$. Suggestion: integral test.

d. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3 + 2n + 5}$

Problem 3. [16; 4 points each part] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you prefer.

_____ **a.** If the terms of a series approach 0, then the series converges.

_____ **b.** If a power series represents a function, then you can find a power series to represent the derivative of the function by differentiating each term, that is,

$$\text{if } f(x) = \sum_{n=0}^{\infty} a_n x^n \text{ then } f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}.$$

_____ **c.** For an alternating series $a_1 - a_2 + a_3 - a_4 + \cdots$, if $\lim_{n\to\infty} a_n = 0$, then the series converges.

_____ **d.** For an alternating series $a_1 - a_2 + a_3 - a_4 + \cdots$, if $\lim_{n\to\infty} a_n \neq 0$, then the series diverges.

Problem 4. [16] On power series. Consider the power series

$$\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} \frac{1}{n2^n} x^n = \frac{1}{2}x + \frac{1}{8}x^2 + \frac{1}{24}x^3 + \cdots + \frac{1}{n2^n}x^n + \cdots$$

Determine the radius of convergence, r, for this series.

Problem 5. [16] For the function $f(x) = \arcsin x$, determine the terms of the power series up to x^3, that is, find a_0 through a_3 in the power series $a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$. To save you time, here are the first few derivatives of $\arcsin x$.

$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}$
$\dfrac{1}{\sqrt{1-x^2}}$	$\dfrac{x}{(1-x^2)^{3/2}}$	$\dfrac{2x^2+1}{(1-x^2)^{5/2}}$	$\dfrac{6x^3+9x}{(1-x^2)^{7/2}}$

#1.[12]	
#2.[40]	
#3.[16]	
#4.[16]	
#5.[16]	
Total	