



Name: _____

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Math 130 Linear Algebra

First Test
27 Oct 2006

You may refer to one sheet of notes on this test, and you may use a calculator. You may leave your answers as expressions such as $\binom{8}{4} \frac{e^{1/3}}{\sqrt{2\pi}}$ if you like. Points for each problem are in square brackets.

Problem 1. [15] The following matrix describes a system of linear equations in five unknowns— v, w, x, y, z . What is the general solution to this system?

$$\left[\begin{array}{ccccc|c} 1 & 5 & 0 & 2 & -2 & 4 \\ 0 & 1 & 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 1 & 7 & -2 \end{array} \right]$$

Problem 2. [10] Recall that a square matrix A is symmetric if it equals its own transpose, that is, $A^T = A$. Prove that for every square matrix B , it is the case that $A = B + B^T$ is a symmetric matrix.

Problem 3. [12; 6 points each part] Let \mathbf{x} be an n -vector.

- Is it possible for the dot product $\mathbf{x} \cdot \mathbf{x}$ to be negative? Explain.
- If $\mathbf{x} \cdot \mathbf{x} = 0$, what is \mathbf{x} ? Explain.

Problem 4. [12] Compute the inverse of the 3×3 matrix A by reducing the 3×6 matrix $[A|I]$ to reduced echelon form. (If it turns out the inverse doesn't exist, explain why you concluded that.)

$$\begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$$

Problem 5. [12] In this problem you will consider permutations of the set $S = \{1, 2, 3, 4, 5, 6, 7\}$.

- [6] How many inversions does the permutation 7531246 have?
- [3] Give an example of an odd permutation of S .
- [3] Give an example of an even permutation of S .

Problem 6. [12] Give a geometric description of the matrix transformation $f : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ defined by $f(\mathbf{u}) = A\mathbf{u}$ when $A = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$.

Problem 7. [12] Compute the following determinant. (Use whatever method you like, but show all your work.)

$$\begin{vmatrix} 5 & 3 & 1 \\ 0 & -2 & 1 \\ 4 & 1 & 2 \end{vmatrix}$$

Problem 8. [15] Prove that if A and B are invertible square matrices of the same size, then AB is also invertible.