CLARK

Name: ___________________________

Mailbox number:___________________

Math 130 Linear Algebra

First Test

5 Oct 2009

Problem 1. [10; 5 points each part] On systems of linear equations and their matrices.

a. Exhibit a system of 3 equations in 5 unknowns whose general solution is

$$(v, w, x, y, z) = (4 - 3x + 2z, x - z, x, z - 1, z)$$

where x and z may be any real numbers.

b. Write the augmented matrix for the system of equations that you gave in part **a**.

Problem 2. [25] Computations. Show your work for credit.

a. [10] Compute the inverse of the matrix A or show that the inverse doesn't exist.

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$$

b. [10] Compute the determinant of the matrix B.

$$|B| = \begin{vmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 3 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 1 & 8 & 1 \end{vmatrix}$$

c. [5] Compute the dot product $\mathbf{v} \cdot \mathbf{w}$ of $\mathbf{v} = (2, 2, -1)$ and $\mathbf{w} = (5, 3, 2)$.

Problem 3. [12; 4 points each part] In this problem you will consider the permutation 23541 of the set $S = \{1, 2, 3, 4, 5\}$.

a. Write the 5×5 permutation matrix for the permuation 23541.

b. Identify the inversions of the permutation 23541.

c. Is 23541 an even permutation or an odd permutation?

Problem 4. [10] Explain why it is the case that when an $n \times n$ matrix A has all zeros in some row, then A has no inverse. Your explanation should be valid for all values of n, not just $n = 2$ or $n = 3$.

Problem 5. [10] On linear transformations of the plane.

a. [6] Give a geometric description of the matrix transformation $f : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $f(\mathbf{u}) = A\mathbf{u}$ when $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$.

b. [4] Does this tranformation preserve distances?

Problem 6. [16] A square matrix $A = [a_{ij}]$ is said to be *skew symmetric* if $A^T = -A$. In other words, for all indices i and j, $a_{ij} = -a_{ji}$. For example,

$$\begin{bmatrix} 0 & 3 & 5 \\ -3 & 0 & -7 \\ -5 & 7 & 0 \end{bmatrix}$$

is a 3×3 skew symmetric matrix.

Prove that if A is a square matrix, then $B = A - A^T$ is a skew symmetric matrix.

Problem 7. [18; 3 points each part] True or false. Just write the word "true" or the word "false". If it's not clear to you which it is, explain; otherwise no explanation is necessary.

_______ **a.** Dot product is commutative: $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$.

_______ **b.** Matrix multiplication is commutative: $AB = BA$.

_______ **c.** Matrix multiplication distributes over addition: $A(B + C) = AB + AC$.

_______ **d.** Matrix inversion distributes over addition: $(A + B)^{-1} = A^{-1} + B^{-1}$.

_______ **e.** Matrix inversion distributes over multiplcation: $(AB)^{-1} = A^{-1}B^{-1}$.

_______ **f.** Matrix inversion satisfies $(A^{-1})^{-1} = A$.

#1.[10]	
#2.[25]	
#3.[12]	
#4.[10]	
#5.[10]	
#6.[16]	
#7.[18]	
Total	