Math 130 Linear Algebra. Selected answers from section 4.1.

23. Show the following

a. $\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = 1$.

In other words, prove that the standard unit vectors actually are unit vectors. There's nothing to it.

$$\mathbf{i}\cdot\mathbf{i} = (1,0)\cdot(1,0) = 1^2 + 0^2 = 1$$

$$\mathbf{j}\cdot\mathbf{j} = (0,1)\cdot(0,1) = 0^2 + 1^2 = 1$$

b. $\mathbf{i}\cdot\mathbf{j} = 0$.

In other words, prove $\mathbf{i} \perp \mathbf{j}$.

$$\mathbf{i}\cdot\mathbf{j} = (1,0)\cdot(0,1) = 0 + 0^2 = 0$$

24. Which of these vectors $\mathbf{u}_1 = (1,2)$, $\mathbf{u}_2 = (0,1)$, $\mathbf{u}_3 = (-2,-4)$, $\mathbf{u}_4 = (-2,1)$, $\mathbf{u}_5 = (2,4)$, and $\mathbf{u}_6 = (-6,3)$ are

a. orthogonal? You might draw these vectors to guess which are orthogonal, then check you guesses algebraically by seeing which inner products are zero, $\mathbf{v}\cdot\mathbf{w} = 0$. You'll find each of $\mathbf{u}_1$, $\mathbf{u}_3$, and $\mathbf{u}_5$ orthogonal to each of $\mathbf{u}_4$ and $\mathbf{u}_6$.

b. in the same direction? Which ones are positive multiples of the others? Or, you could compute the unit vector for each and see which have the same unit vector. You'll find $\mathbf{u}_1$ and $\mathbf{u}_5$ point in the same direction; also $\mathbf{u}_4$ and $\mathbf{u}_6$ point in the same direction

c. in the opposite direction? Which are negative multiples of the others? $\mathbf{u}_3$ is in the opposite direction of $\mathbf{u}_1$ and $\mathbf{u}_5$.

28a. Write $3\mathbf{i} - 2\mathbf{j}$ as a 2×1 matrix.

$$3\mathbf{i} - 2\mathbf{j} = 3\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

T.5. Show that if $\mathbf{x}$ is a nonzero vector, then

$$\mathbf{u} = \frac{1}{\|\mathbf{x}\|}\,\mathbf{x}$$

is a uanait vector in the direction of $\mathbf{x}$.

First, show that $\mathbf{u}$ is a unit vector. Since $\|\mathbf{x}\|$ is a scalar, therefore

$$\left\| \frac{1}{\|\mathbf{x}\|}\,\mathbf{x} \right\| = \frac{1}{\|\mathbf{x}\|}\,\|\mathbf{x}\|$$

which equals 1.

Next, $\mathbf{x}$ is in the same direction as $\mathbf{u}$ since it's a positive multiple of $\mathbf{u}$, namely, $\mathbf{x} = \|\mathbf{x}\|\,\mathbf{u}$.

T.7. Prove the properties of dot product. Here are a couple of the proofs.

Commutativity: $\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$.

Let $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$. Then

$$\mathbf{u}\cdot\mathbf{v} = u_1v_1 + u_2v_2 = v_1u_1 + v_2u_2 = \mathbf{v}\cdot\mathbf{u}.$$

Associativity with scalar product: $(c\mathbf{u})\cdot\mathbf{v} = \mathbf{u}\cdot(c\mathbf{v}) = c(\mathbf{u}\cdot\mathbf{v})$.

The left expression is

$$(c\mathbf{u})\cdot\mathbf{v} = (cu_1, cu_2)\cdot(v_1, v_2) = cu_1v_1 + cu_2v_2.$$

The middle expression is

$$\mathbf{u}\cdot(c\mathbf{v}) = (u_1, u_2)\cdot(cv_1, cv_2) = u_1cv_1 + u_2cv_2.$$

The right expression is

$$c(\mathbf{u}\cdot\mathbf{v}) = c(u_1v_1 + u_2v_2).$$

These are all equal.