Math 130 Linear Algebra. Selected answers 4.2.

8a. Determine the vector in $\mathbf{R}^3$ whose tail is at the point $(2,3,-1)$ and whose head is at the point $(0,0,2)$.

This is usually called the *displacement vector* between the two points. Just subtract the tail from the head. You'll get $(-2,-3,3)$. You could write that as a column vector if you prefer.

10a. Find the length of the vector $(1,2,-3)$.

Compute the square root of the sum of the squares of the coordinates.

$$\sqrt{1^2+2^2+(-3)^2} = \sqrt{14}.$$

12a. Find the distance between the two points $(1,-1,2)$ and $(3,0,2)$.

Compute the square root of the sum of the squares of the differences between corresponding coordinates.

$$\sqrt{(3-1)^2+(0+1)^2+(2-2)^2} = \sqrt{5}.$$

14. Is the vector $(2,-2,3)$ a linear combination of the vectors $(1,2,-3)$, $(-1,1,1)$, and $(-1,4,-1)$?

This exercise asks the question: do there exist real numbers x, y, and z so that $x(1,2,-3)+y(-1,1,1)+z(-1,4,-1)=(2,-2,3)$? This single vector equation corresponds to a system of linear equations.

$$\begin{array}{rrrcr} x & -y & -z & = & 2 \\ 2x & +y & +4z & = & -2 \\ -3x & +y & -z & = & 3 \end{array}$$

You can use any of the usual methods to solve this system. You'll find that it's inconsistent, that is, there are no solutions. Therefore, the first vector is *not* a linear combination of the other three vectors.

21a. Find the cosine of the angle between the two vectors $\mathbf{u}=(2,3,1)$ and $\mathbf{w}=\mathbf{b}=(3,-2,0)$.

We can find the cosine without resorting to a calculator, but to find the actual angle in general, we'd need to evaluate an inverse cosine. It turns out for this problem, we could find the angle without using a calculator. Use the formula

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

which in this case gives

$$\begin{aligned} \cos\theta &= \frac{(2,3,1)\cdot(3,-2,0)}{\|(2,3,1)\|\,\|(3,-2,0)\|} \\ &= \frac{0}{\|(2,3,1)\|\,\|(3,-2,0)\|} = 0 \end{aligned}$$

(Since the cosine of the angle is 0, therefore the angle is 90°.)

23. Which of the vectors

$$\begin{array}{ll} \mathbf{u}_1=(4,2,6,-8), & \mathbf{u}_2=(-2,3,-1,-1), \\ \mathbf{u}_3=(-2,-1,-3,4), & \mathbf{u}_4=(1,0,0,2), \\ \mathbf{u}_5=(2,3,4,-4), \text{ and} & \mathbf{u}_6=(0,-3,1,0) \end{array}$$

are (a) orthogonal, (b) parallel, and (c) in the same direction?

Questions (b) and (c) are easier to answer and may help in answering (a). To be in the same direction, one vector is a positive multiple of the other, but to be parallel, all that's required is that one vector be a nonzero multiple of the other. By inspection, you can see that only $\mathbf{u}_1$ and $\mathbf{u}_3$ are parallel, but they're in the opposite direction.

Now to determine which vectors are orthogonal to which other vectors, and that means finding when their dot products are 0. Is $\mathbf{u}_6$ orthogonal to any of the other vectors? (I'm starting with that because two of its coordinates are 0 and so the computations are easier.) Yes. $\mathbf{u}_6 \perp \mathbf{u}_1, \mathbf{u}_3, \mathbf{u}_4$. Checking $\mathbf{u}_5$ next, I see it's not orthogonal to any of the rest. Then $\mathbf{u}_4 \perp \mathbf{u}_6$ only. Next $\mathbf{u}_2 \perp \mathbf{u}_1, \mathbf{u}_3$.

ML.2b. Determine the norm or length of $\mathbf{v} = \begin{bmatrix} 0 \\ 4 \\ -3 \\ 0 \end{bmatrix}$

```
>> v=[0;4;-3;0]
v =
     0
     4
    -3
     0
>> norm(v)
ans =
     5
```

ML.3b. Determine the distance between $\mathbf{u}=(2,0,0,1)$ and $\mathbf{v}=(2,5,-1,3)$

```
>> norm([2 0 0 1]-[2,5,-2,3])
ans =
    5.7446
```

ML.5b. Determine the dot product of $\mathbf{u}=(3,-1,0,2)$ and $\mathbf{v}=(-1,2,-5,-3)$.

```
>> dot([3 -1 0 2],[-1 2 -5 -3])
ans =
   -11
```

ML.8b. Find the angle between the vectors $\mathbf{u}=(2,2,-1)$ and $\mathbf{v}=(2,0,1)$

```
>> u=[2 2 -1]
u =
     2     2    -1
>> v=[2 0 1]
v =
     2     0     1
>> arccostheta=dot(u,v)/(norm(u)*norm(v))
arccostheta =
    0.4472
>> theta = acos(arccostheta)
theta =
    1.1071
>> thetadegrees = theta * 180 / pi
thetadegrees =
   63.4349
```