CLARK

Name: ______________________________

Math 131 Multivariate Calculus

Quiz

17 Mar 2014

Problem 1. On arclength. The path $\mathbf{x} : \mathbf{R} \to \mathbf{R}^2$ given by

$$\mathbf{x}(t) = (x(t), y(t)) = (\cos^3 t, \sin^3 t)$$

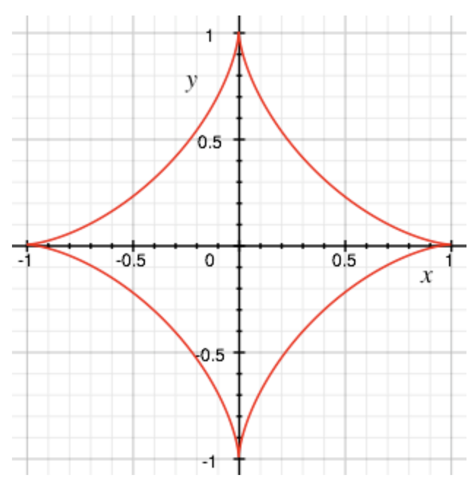
describes an astroid, that is, a star shaped figure. For t in the interval $[0, \frac{\pi}{2}]$, one fourth of the astroid is described. It is easy to see that

$$\mathbf{x}' = (-3 \sin t\, \cos^2 t, 3 \sin^2 t\, \cos t),$$

and

$$\|\mathbf{x}'\| = 3 \sin t\, \cos t = \tfrac{3}{2} \sin 2t.$$

Using that information, determine the arclength of this one-fourth astroid.



Problem 2. On flow lines. Recall that a flow line for a vector field $\mathbf{F}$ is a path $\mathbf{x}$ such that the velocity along the path is a vector in the vector field, that is, $\mathbf{x}'(t) = \mathbf{F}(\mathbf{x}(t))$. Verify that the path $\mathbf{x}(t) = (\sin t, \cos t, 2t)$ is a flow line for the vector field $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + 2\mathbf{k} = (y, -x, 2)$.

Problem 3. On divergence and curl of vector fields.

a. Give an example of a vector field with a nonzero divergence, and compute its divergence.

b. Give an example of a vector field with a nonzero curl, and compute its curl.