CLARK UNIVERSITY

Math 217/Econ 360

Second Test Extra Answers

Nov 2014

Answers for the extra credit problems. 4 points for each.

8. The joint density function of X and Y is given by $f(x, y) = xe^{-x(y+1)}$ for positive x and y, and 0 otherwise. Write down a double integral which gives the probability of the event E in which $|X - Y| \leq 1$. (Don't evaluate the integral.)

The event $|X - Y| \leq 1$ describes a diagonal strip above and below the line $y = x$. We're only interested in the part of that strip in the first quadrant. You can slice it either with horizontal or vertical lines, but either way you'll use the sum of two integrals. Here are vertical slices.

$$\int_{x=0}^{1} \int_{y=0}^{x+1} xe^{-x(y+1)}\, dy\, dx + \int_{x=1}^{\infty} \int_{y=x-1}^{x+1} xe^{-x(y+1)}\, dy\, dx$$

The reason you need two integrals is because the nested integral has different limits of integration depending on the value of x.

9. The gross weekly sales at a certain restaurant are normally distributed with a mean of \$5000. If about 20% of the weeks gross less than \$3000, then what is the standard deviation of gross weekly sales? About what percentage of the weeks gross more than \$8000?

30% below the mean corresponds to \$2000 below the mean. By the normal distribution table, 30% is about 0.84 standard deviations. If \$2000 is 0.84 standard deviations then 1 standard deviation is about \$2380. \$8000 is \$3000 above the mean, or 1.26 standard deviations above the mean, and below that is about 89.6%, so above that is about 10.4%.

10. Let the joint density function of X and Y be $f(x, y) = 24xy$ on the triangle where $0 \leq x$, $0 \leq y$, and $x + y \leq 1$ (and f is 0 otherwise.) Determine the marginal density function $f_X(x)$ for X. Compute the expectation $E(X)$.

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy = \int_0^{1-x} 24xy\, dy = 12xy^2 \Big|_{y=0}^{1-x} = 12x(1-x)^2$$

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f_X(x)\, dx = \int_0^1 12x^2(1-x)^2\, dx \\ &= \int_0^1 12(x^2 - 2x^3 + x^4)\, dx = 12(\tfrac{1}{3}x^3 - \tfrac{1}{2}x^4 + \tfrac{1}{5}x^5)\Big|_0^1 = \tfrac{2}{5} \end{aligned}$$