Math 217/Econ 360

Sample Second Test

Nov 2014

(This sample test is longer than the actual test will be.)
You may refer to one sheet of notes on this test, the summary table of distributions below, the table for the normal distribution, and you may use a calculator. Show your work for credit. You may leave your answers as expressions such as $\binom{8}{4}\frac{e^{1/3}}{\sqrt{2\pi}}$ if you like.

1. A point is chosen at random on a line segment of length 12. Determine the probability that the ratio of the shorter segment to the longer segment is less than 1/4.

2. A dart is thrown at a circular dartboard of radius 10 inches. Assume that the location that the dart lands is a uniform continuous distribution, that is, the probability that it lands in a region of the dartboard is proportional to the area of that region.

a. What is the probability that the dart falls within 5 inches of the center of the target?

b. What is the probability that the dart falls within 3 inches of the edge of the target?

3. For some constant c the continuous random variable X has the probability density function $f(x) = cx^4$ on the interval $[0, 2]$ and 0 elsewhere.

a. Determine the value of c.

b. Determine the value of $E(X)$.

c. Determine the value of $\mathrm{Var}(E)$.

4. A randomly chosen IQ test taker obtains a score that is approximately a normal random variable with mean 100 and standard deviation 15. What is the probability that the score of such a person is

a. more than 125?

b. between 90 and 110?

5. The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Ms. Jones buys a used radio, what is the probability that it will be working after after 8 years?

6. Suppose that the joint probability density function $f(x, y)$ for the random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2) & \text{if } 0 \le x \le 1, 0 \le y \le 1, \\ 0 & \text{otherwise} \end{cases}$$

a. Evaluate the probability $P(X \le \frac{1}{2} \text{ and } Y \le \frac{1}{2})$.

b. Determine the marginal density function $f_X(x, y)$.

c. Determine the conditional density function $f_{Y|X}(y|x)$.

7. Suppose that we model the arrival of airport shuttle buses at a terminal by a Poisson process where the rate of arrivals is $\lambda = 6$ buses per hour.

a. Determine the probability that a person has to wait less than 10 minutes.

b. Determine the probability that exactly 6 buses arrive in one hour.

8. Choose a number U from the interval $[0, 1]$ uniformly. Find the cumulative distribution function $F_Y(y)$ and the probablility density function $f_Y(y)$ for the random variable $Y = 1/(U + 1)$.

9. Suppose that X and Y are independent random variables each with Gamma distributions having parameters λ and r. Let $Z = X + Y$. Write down the integral which gives the density function for Z. (Do not evaluate the integral.)

Distribution	Type	Mass/density function $f(x)$	Mean μ	Variance σ^2
$\textsc{Uniform}(n)$	D	$1/n,$ for $x = 1, 2, \ldots, n$	$(n+1)/2$	$(n^2-1)/12$
$\textsc{Uniform}(a, b)$	C	$\frac{1}{b-a},$ for $x \in [a, b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$\textsc{Bernoulli}(p)$	D	$f(0) = 1-p, f(1) = p$	p	$p(1-p)$
$\textsc{Binomial}(n, p)$	D	$\binom{n}{x}p^x(1-p)^{n-x},$ for $x = 0, 1, \ldots, n$	np	npq
$\textsc{Geometric}(p)$	D	$q^{x-1}p,$ for $x = 1, 2, \ldots$	$1/p$	$(1-p)/p^2$
$\textsc{NegativeBinomial}(p,r)$	D	$\binom{x-1}{r-1}p^r q^{x-r},$ for $x = r, r+1, \ldots$	r/p	$r(1-p)/p^2$
$\textsc{Hypergeometric}(N,M,n)$	D	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}},$ for $x = 0, 1, \ldots, n$	np	$np(1-p)$
$\textsc{Poisson}(\lambda t)$	D	$\frac{1}{x!}(\lambda t)^x e^{-\lambda t},$ for $x = 0, 1, \ldots$	λt	λt
$\textsc{Exponential}(\lambda)$	C	$\lambda e^{-\lambda x},$ for $x \in [0, \infty)$	$1/\lambda$	$1/\lambda^2$
$\textsc{Gamma}(\lambda, r)$ $\textsc{Gamma}(\alpha, \beta)$	C	$\frac{1}{\Gamma(r)}\lambda^r x^{r-1} e^{-\lambda x}$ $= \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)},$ for $x \in [0, \infty)$	r/λ $= \alpha\beta$	r/λ^2 $= a\beta^2$
$\textsc{Beta}(\alpha, \beta)$	C	$\frac{1}{\mathrm{B}(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$ for $0 \le x \le 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
$\textsc{Normal}(\mu, \sigma^2)$	C	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$ for $x \in \mathbf{R}$	μ	σ^2