CLARK UNIVERSITY 1887

Conditional distributions
Math 217 Probability and Statistics

Prof. D. Joyce, Fall 2014

Suppose you have joint distributions X and Y and denote their joint cumulative distribution function by $F(x, y)$ and their joint probability mass or density function by $f(x, y)$. Their marginal cumulative distribution functions are $F_X(x)$ and $F_Y(y)$ and their probability functions are $f_X(x)$ and $f_Y(y)$ as we saw before.

There are also conditional distribution and probability functions.

The conditional cumulative distribution function for X given that Y has the value y is denoted in various ways. Our text denotes it $F_{X|Y}(x|y)$. Likewise, the corresponding conditional probability mass or density function is denoted $f_{X|Y}(x|y)$. There are also conditional functions for Y given that X has a value x.

As you would expect, if X and Y are independent, then the conditional probability functions are the same as the marginal functions:

$$f_{X|Y}(x|y) = f_X(x).$$

The definition in the discrete case. Here, X and Y are joint discrete random variables. The conditional cumulative distribution function $F_{X|Y}(x|y)$ is

$$\begin{aligned} F_{X|Y}(x|y) &= P(X{\le}x|Y{=}y) \\ &= \frac{1}{f_Y(y)} \sum_{t \le x} f(t, y) \end{aligned}$$

The conditional probability mass function $f_{X|Y}(x|y)$ for X given Y has the value y is

$$f_{X|Y}(x|y) = P(X{=}X|Y{=}y) = \frac{f(x, y)}{f_Y(y)}.$$

The definition in the continuous case. For the continuous case the conditional cumulative distribution function $F_{X|Y}(x|y)$ is

$$\begin{aligned} F_{X|Y}(x|y) &= P(X{\le}x|Y{=}y) \\ &= \frac{1}{f_Y(y)} \int_{-\infty}^{x} f(t, y)\, dx \end{aligned}$$

Its derivative with respect to x gives the conditional probability density function $f_{X|Y}(x|y)$ for X given Y has the value y is

$$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)}.$$

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