Math 217, Quiz Answers, Sep 2007

Scores: 9–10 A, 7.5–8 B, 6–7 C. Median 8.

Problem 1. [6 points] Three fair dice are tossed.

a. [1] How many outcomes are in the sample space Ω?

There are $6^3 = 216$.

b. [1] What is the probability that all three turn up sixes?

Thot's one outcome, so it has probability 1/216.

c. [2] What is the probability that all three turn up the same?

This event includes 6 outcomes, so its probability is $\frac{6}{216} = \frac{1}{36}$.

d. [3] Give a one or two sentence proof that if $A \subseteq B$, then $P(A) \leq P(B)$.

The probability $P(A)$ is the sum of the probabilities of the outcomes in A, while $P(B)$ is the sum of the probabilities of the outcomes in B. Since all the outcomes in A are also outcomes in B, therefore $P(B)$ equals $P(A)$ plus the probabilities of all the outcomes in B not in A, and each of those probabilities is nonnegative, so $P(B) \geq P(A)$.

Here's a different argument you could give that doesn't depend on outcomes. Let C be the part of B that doesn't include A. (This difference is usually denoted $B - A$.) Since $A \subseteq B$, therefore $B = A \cup C$. Since A and C are disjoint, therefore $P(B) = P(A) + P(C)$. But $P(C) \geq 0$, hence $P(B) \geq P(A)$.

Problem 2. [4 points] Suppose four fair coins are tossed simultaneously. Determine the probability that of these four coins, exactly two come up heads while the other two come up tails. Show your work, or explain your answer, or both.

There are $2^4 = 16$ outcomes in the sample space. One has 4 heads, 4 have 3 heads and a tail, 6 have two heads and two tails, 4 have one head and 3 tails, and 1 has 4 tails. So the probability of 2 heads and 2 tails is $\frac{6}{16} = \frac{3}{8}$.