Welcome
Math 217 Probability and Statistics
Prof. D. Joyce, Fall 2014

Welcome to the course!

Much of today we'll discuss the administration of the course and discuss what the course covers. The course web page is at `http://math.clarku.edu/~djoyce/ma217/`

We will discuss the general description of the course, its goals and objectives, the syllabus, the textbook, assignments, quizzes, tests, course grade, and various and sundry items.

Discrete uniform probability. For the first week of the course, we'll introduce the concept of probability by examining the case where there are finitely many outcomes to an experiment where they're all equally likely. This will allow us to see what's going on right away. We'll answer some of the questions that come up in this setting by counting things, and that means we'll do a review of basic combinatorics as well.

Figure 1: Symmetric random generators

An example of such a thing is where a fair cubical die is tossed. It will come up with one of the six outcomes 1, 2, 3, 4, 5, or 6, and each with the same probability. Another example is where a fair coin is flipped. It will come up with one of the two outcomes H or T.

Terminology and notation. We'll call the tossing of a die a *trial* or an *experiment*. We should probably reserve the word experiment for a series of trials, but when there's only one trial under consideration, then the experiment is just the trial.

The possible values that can occur when a trial is performed are called the *outcomes* of the trial. The outcomes for rolling a die are 1, 2, 3, 4, 5, and 6, while the outcomes for flipping a coin are H and T.

Usually, we'll introduce a symbol for the outcome of an experiment, such as X, and call this symbol a *random variable*. So, if you flip a coin and an H comes up, then $X = H$, but if a T comes up, then $X = T$. We'll study random variables in detail later in the course.

Let's take the case of tossing a fair die. When it's tossed, all six outcomes are equally probable, so they should all have the same probability. We'll want the sum of all the probabilities of the outcomes to be 1, so for a fair die, each outcome will have probability $\frac{1}{6}$. We'll use a notation like $P(X{=}4) = \frac{1}{6}$, read "the probability that $X = 4$ is equal to $\frac{1}{6}$," to indicate that 4 occurs as an outcome with a probability of $\frac{1}{6}$.

This probability notation is flexible since we can use it to express probabilities other than X having a particular value. For instance $P(X \geq 5)$ expresses the probability that the die comes up with a value greater than or equal to 5, that is to say, either 5 or 6. Since each of 5 and 6 have probabilities of $\frac{1}{6}$, therefore $P(X \geq 5) = \frac{2}{6} = \frac{1}{3}$. Thus, the probability notation allows us to express probabilities of sets of outcomes, not just single outcomes. We'll call these sets of outcomes events. Thus, an *event* is a subset of all of the possible outcomes.

Symmetry leads to uniform probabilities. So far, our two examples are uniform. A fair coin has two sides, and we assign the same probability to each because of the the physical symmetry of the coin. Likewise, we assign the same probability to each of the six sides of a fair die because of symmetry of the die. There are lots of other situations where symmetry implies that the probabilities of the outcomes should be equal. For instance, some of the examples we'll look at involve choosing balls from urns. Say an urn has 5 balls in it. Take out one of the balls at random. There are five possibilities, and each ball has the same probability of being chosen. Thus, we get a uniform probability where the probability of choosing any particular ball is $\frac{1}{5}$.

The symmetry of the situation is required, however, to conclude that the probability is uniform. For instance, if you throw a ball at the moon, there are two outcomes—either it hits the moon or it doesn't. But the two outcomes are not symmetric, and it would be foolish to assume that the probability that your ball hits the moon is $\frac{1}{2}$.

The frequency concept of probability. When there's no symmetry, we can't assume that the probability is uniform. Suppose you bend a coin. The coin is no longer symmetric, so we can't assume that H and T have the same probability. But you can get some idea of what the probability is by performing the experiment of flipping the coin repeatedly. Suppose you flip the coin 1000 times and you get 412 heads and 588 tails. It would be reasonable to conclude that $P(H)$ is near 0.412, and $P(T)$ is near 0.588. However, it would be foolish to believe those are the *exact* probabilities. It is unlikely that next 1000 coin flips will also produce 412 heads and 588 tails.

Most of what we study this semester is the theory of Probability. For that, we assume that the basic probabilities that we use are known. But for some of the situations we will consider, we won't know the basic probabilities and the problem will be to determine somehow by experiment what those probabilities are, and that's the province of Statistics. Thus, the conclusion in the last paragraph that $P(H)$ is near 0.412 is a statement in Statistics rather than a statement in Probability.

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