Math 218 Mathematical Statistics

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27 Feb 2009

Due Monday after spring break. Page 262: 1, 2, 9, 11, 12.

Last meeting. Discussed t-confidence intervals and t-tests.

Today. Inferences on variances.

Inferences on variances. In 7.3, we switch from making inferences about μ to inferences about σ^2, but only in the case that the population distribution is a normal distribution. The statistical tests will all involve the χ^2 distribution.

When the population distribution is a normal $N(\mu, \sigma^2)$ distribution, then the sample variance S^2 is an estimator of the population variance σ^2, furthermore,

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

is a chi square distribution with $n-1$ degrees of freedom. That's the statistic we'll use for confidence intervals and hypothesis tests for σ^2.

Thus, a two-sided confidence interval for σ^2 at the confidence level $1-\alpha$ is

$$\left[\frac{(n-1)S^2}{\chi^2_{n-1,\frac{\alpha}{2}}}, \frac{(n-1)S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}\right],$$

while a lower one-sided confidence interval at the same confidence level is

$$\left[\frac{(n-1)S^2}{\chi^2_{n-1,\alpha}}, \infty\right),$$

and an upper one-sided confidence interval is

$$\left(-\infty, \frac{(n-1)S^2}{\chi^2_{n-1,1-\alpha}}\right].$$

These confidence intervals directly give corresponding hypothesis tests.

For a two-sided hypothesis test with $H_0 : \sigma^2 = \sigma_0^2$, reject H_0 if σ_0^2 is not in the first interval. For a lower one-sided hypothesis test with $H_0 : \sigma^2 \geq \sigma_0^2$, reject H_0 if σ_0^2 is not in the second interval. And for an upper one-sided hypothesis test with $H_0 : \sigma^2 \leq \sigma_0^2$, reject H_0 if σ_0^2 is not in the third interval.