Review of *Mathematics of Physics and Engineering*
Authors: *Edward K. Blum and Sergey V. Lototsky*
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1 Overview

This text is designed for a course typically given in the junior or senior year, addressed to scientists (especially physicists and chemists) and engineers, detailing various topics in mathematics that arise in those disciplines. Generally these courses assume some knowledge of physics, and a mathematical background that consists of an introductory sequence through multivariate calculus, and first courses in linear algebra and differential equations. Major topics typically include vector analysis, complex analysis, Fourier analysis, and partial differential equations. The popular and/or classic texts that come to mind in this category include, for example, Arfken and Weber's Mathematical Methods for Physicists, Boas's Mathematical Methods in the Physical Sciences, or Kreyszig's Advanced Engineering Mathematics. Although it plays a similar role, Blum and Lototsky takes an unconventional approach. The topics are more focussed and many are out of the scope of these other textbooks. In a nutshell, it puts much greater emphasis on the interplay between mathematics and physics, and goes to great lengths to involve the reader. This is discussed at some length in the last section of this review.

2 Summary of Contents

Chapter 1: Euclidean Geometry and Vectors
The foci of this chapter are elementary notions of geometry, vectors, and kinematics. The chapter begins with a quick review of Euclidean geometry. The idea of a reference frame is also introduced, and is returned to again and again over the next 3 chapters. Using this solid conceptual basis, vectors and associated operations such as the inner and cross products are defined and discussed. Curves in $\mathbf{R}^3$ are introduced as vector functions of a scalar variable, and the description of the curve's points and corresponding tangents is given via the Frenet formulas. The chapter ends with the definition of velocity and acceleration, giving basic properties and expressing them in different coordinate systems.

Chapter 2: Vector Analysis and Classical and Relativistic Mechanics
The subject of dynamics is taken up in earnest. Newton's Laws of motion are formulated and re-expressed in different reference frames, both inertial and non-inertail. A consequence of the latter, in the context of uniformly rotating frames, are clear expositions of interesting phenomena such as Foucault's pendulum and the Coriolis force. Transformations between generally accelerated frames also provide a foundation for the later treatment (in the same chapter) of general relativity.

Systems of point masses, both rigid and non-rigid are discussed, along with the requisite concepts of angular momentum and moments of inertia. The Euler-Lagrange and Hamiltonian formalisms of classical physics receive a succinct treatment. The development then moves on to *relativistic* mechanics. Enough of the theory of special relativity is explained to obtain results like $E = mc^2$ and the Lorentz-Fitzgerald contraction. The Einstein field equations for general relativity are then postulated; this is one (very understandable) instance in which the physical origin of the equations is not explained in detail. While it is nice for the reader to see the equations even without extensive explanation, and the explanations given are by in large correct, there are some inaccuracies, which are explained in the "Evaluation" section. The section is nevertheless a good quick (!) introduction to general relativity, going so far as to give the Schwarzschild solution and some of its most important consequences, including part of the theory of black holes.

Chapter 3: Vector Analysis and Classical Electromagnetic Theory
The mathematical thread running through this chapter is, not surprisingly given its title, classical vector field theory. It begins with basic definitions about vector functions, scalar and vector fields, and the gradient. It introduces line and surface integrals, and the divergence and curl operators. An appealing feature is the use of elementary notions of measurability and limits (still within a very concrete framework) to write *coordinate-free* definitions of div and curl. It is also explained how these operators can be expressed in arbitrary orthogonal curvilinear coordinate systems. Next the central integral theorems of vector analysis are presented: the theorems of Green (no relation!), Stokes, and Gauss. The physical intuition underlying these theorems is explained and, as with most of the major theorems in the text, the proofs are given as exercises. The mention of the generalized version of Stokes's Theorem from differential geometry, for which much of the intuition is established in this (and the previous) chapter, is a nice touch. One application of Gauss's theorem is given via an introduction to potential theory (in particular, properties of Laplace's and Poisson's equations). The chapter culminates in a section on Maxwell's equations. They are derived from physical principles (e.g., Coulomb's Law and Ampère's Law) as well as the earlier mathematical results in the chapter (e.g., Gauss's Theorem). Some static solutions are presented, including those for fields surrounding electric and magnetic dipoles; dynamic solutions are put off until Chapter 6. Maxwell's equations in material media are also given (up to notation, the same as those that hold in the vacuum); the main emphasis here is physical, but it seems the authors are not letting any opportunity pass to point out the uniformity of the mathematical framework. The observation that the vector potential is not unique, thus hinting at gauge invariance (which is discussed in a little more detail in chapter 6) is a welcome feature.

Chapter 4: Elements of Complex Analysis
This chapter largely leaves physics and engineering behind for a time, and gives a fairly standard basic treatment of functions of a complex variable. The definitions of complex variables and the complex plane, along with some brief history, followed by an example using complex numbers to analyze AC circuits[1], all provide good motivation for exploring the theory. Functions of a complex variable and analytic functions are defined. An example is given to motivate the Cauchy-Riemann equations, and again the reader is invited to participate in the proof that they are an equivalent criterion for differentiability. This is followed by Cauchy's Integral Theorem and Integral Formula. Cauchy's Integral Theorem is used, among other things, to prove the Fundamental Theorem of Al-

[1] Notation alert for a sizable fraction of SIGACT readers: here "AC" really stands for "alternating current"!

gebra. Conformal mappings are explained and used as a tool for the analysis of Laplace's equation in two dimensions. We proceed to power series, convergence, and Taylor series as a characterization of analyticity. Next come the Laurent series, various types of singularities including poles, the Residue Theorem and some fine examples of residue integration. There is brief mention of branch points, but branch cuts and Riemann surfaces are not discussed. There is a section on power series solutions of ordinary differential equations, the emphasis being on complex solutions near singular points.

Chapter 5: Elements of Fourier Analysis
The chapter first lays foundations, then proceeds (more or less) to successively more general methods. Basic definitions and properties of Fourier series and coefficients are given, along with a brief exposition of Bessel's Inequality and Parseval's Identity. Point-wise convergence is contrasted with uniform convergence, for which the Weierstrass M-test is given, as is a sufficient condition for a function to have a Fourier series. Motivational and historical notes here provide a good orientation for the reader; this includes the discussion of the connection between Fourier series and signal processing, and a physical interpretation of Parseval's Identity. After some applications to ordinary differential equations, the text moves on to the (continuous) Fourier transform. This is investigated in much the same spirit and using similar methodologies to that used for the Fourier series. An introduction to the discrete Fourier transform includes well-motivated brief introductions to the Dirac δ-function and the fast Fourier transform. The final section of the chapter investigates the Laplace transform, including a section giving applications to system theory, a good illustration of the chapter's techniques.

Chapter 6: Partial Differential Equations of Mathematical Physics
This is by far the longest chapter, and taking up almost a quarter of the book it covers quite a bit of ground in the field of classical solutions of partial differential equations (PDEs). The first section uses simple examples to illustrate the more general solution techniques that follow: variation of parameters is used to solve the transport equation, and Fourier analysis and separation of variables for the heat and wave equations. Some of the physical derivations of these equations are included. There follows an introduction to the general theory of PDEs. This includes some methods of classification, the method of characteristics, variation of parameters, and separation of variables. The techniques are given adequate detail, are nicely summarized and illuminating examples are given, although it is a little surprising that the phrase "variation of parameters" is left unexplained. A long section considers various classical PDEs. These include the telegraph, Helmholtz, wave and Maxwell's equations, as well as some equations of fluid mechanics including Navier-Stokes. For Maxwell's equations, there is some discussion of gauge invariance and gauge fixing, and the propagation of electromagnetic waves is derived. The next section turns to equations of quantum mechanics, focussing on Schrödinger's equation. As in the case of general relativity, adequate justification can hardly be expected, but there is a very good sketch of the history and results leading up to the equation. The postulates of quantum mechanics are stated and some solutions of Schrödinger's equation are given, one quite simple (the harmonic oscillator) and one not so simple (the hydrogen atom). In the latter case, building on the techniques developed earlier, all energy levels are computed in the non-relativistic approximation. The other "equation of quantum mechanics" that is considered is the Dirac equation. This includes Dirac's magical derivation of the equation, some explanation of the nature of intrinsic spin, and a brief hint as to the existence of

anti-matter. Included in the section on quantum mechanics is a sub-section on quantum computing. The material includes, early on, the Deutsch algorithm (for evaluating $f(0) \oplus f(1)$ with only one query to f, stated here as an exercise), with a later hint at its generalization, the Deutsch-Josza algorithm. The basic ideas (qubits, universal sets of quantum gates, entanglement, quantum algorithms) are defined and discussed. There are brief discussions of Grover's and Shor's algorithms, but no technical details. The final section of Chapter 6 is a survey of numerical methods for PDEs. The numerical quadrature problem is defined, and explicit and implicit methods and stability are illustrated via ordinary differential equations. Finite difference methods are applied to the heat, wave and Poisson equations. The chapter concludes with an introduction to the finite element method.

Chapter 7: Further Developments
This is a set of problems, some of them quite substantial, that extend material in the text proper. For example, one problem leads the reader through the calculation, drawing on the Schwarzschild solution of Chapter 2, of the precession of the perihelion of Mercury. Another is an analysis of the Michelson-Morley experiment. There is a problem that entails an introduction to quaternions, and another that works through the proof of point-wise convergence of Fourier series. There is a long exercise on the 1D Sturm-Liouville problem, and a shorter one on solitons in the Korteweg-deVries equation (this is a nonlinear PDE that describes shallow waves, and the soliton phenomenon has many important analogs in other areas of physics). There are many others of a similar nature.

Chapter 8 is an appendix that includes, most notably, review material on linear algebra, ordinary differential equations, and tensors.

3 Evaluation and Opinion

Considering the sheer bulk of other volumes in this category (e.g., Kreyszig's text weighs in at 1248 pages), it is refreshing to encounter a book such as Blum and Lotosky, which isn't any harder to lift than the average novel. Part of the reason is that it makes no attempt to be encyclopedic, being designed for a one-semester course. But far more importantly, the book largely motivates and outlines the subject, leaving it up to the reader to provide much of the substance. Exercises are tightly integrated with the text, with the proofs of major theorems (in part or in their entirety), and significant calculations, being stated as exercises for the reader, generally given with ample hints. Those who work through all the exercises are bound to feel more like a *participant* than a passive reader, and indeed might almost end with the impression that they contributed to the writing of the book. If this isn't sound mathematical pedagogy, I don't know what is.

It is also distinguished by the tight integration of mathematics and its applications (largely physics). Many physical principles are introduced virtually from scratch to derive the relevant mathematics, and the mathematics in turn is used to derive physical results, so often it reads more like physics than mathematics. Very important physical ideas (e.g., relativity and quantum mechanics) are included that are usually omitted in conventional books in this category.

There are, on the other hand, a number of inaccuracies, shortcomings or quibbles I feel compelled to point out:

- It is stated on page 106 that the Einstein equations describe "the relation between the metric tensor and the gravitational field..." Actually, the metric *replaces* the gravitational field; the

Einstein equations give the relation between the metric and the *matter* fields as embodied in the energy-momentum tensor. (In a future edition the authors may consider including Wheeler's incomparable aphorism, "matter tells space-time how to curve, and curved space tells matter how to move;" this description is indeed enumerated in somewhat greater detail at the bottom of page 110.) Later, on page 118, in the course of deriving the (correct!) gravitational red-shift, it is erroneously stated that a certain photon of frequency ν has mass $h\nu/c^2$. All photons have zero mass. What is true is that a massive particle can *lose* a mass of $h\nu/c^2$ by emitting such a photon. This fact can be used, together with the principle of equivalence, to give a correct argument (as is done in some books on general relativity [2]).

- In Chapter 3, there is a misrepresentation that ought to be addressed, even though it is only mentioned in passing. On page 178 (and again on page 351) the authors describe Yang-Mills theory as a "quantum-theoretic analog of Maxwell's equations." In fact, Yang-Mills theory can be formulated classically. Like Maxwell's theory, the classical Yang-Mills theory must be quantized to obtain an analog of quantum electrodynamics. It is far more accurate to say that the Yang-Mills equations are analogs of Maxwell's equations because the gauge group, $U(1)$ in the case of Maxwell, becomes non-abelian (and, with a little more effort, this could be stated in a more accessible manner to the intended readers of the book).

- In Chapter 4, I felt a good opportunity was missed in not including the application of conformal mappings to simple problems of fluid flow and/or electrostatics, which would have fit very nicely with the overall aims of the the text.

- In Chapter 5, the section on system theory does not contain any explanation of the terminology or motivation – quite at odds with the rest of the book.

- The section on quantum computing in Chapter 6 is a reasonable sketch, but it is oddly placed, since (insofar as it is covered here) it has no direct relationship with PDEs.

- Generally, there could be many more figures, and the language is occasionally a bit stilted.

Despite these problems, none of which are major, this is a good text. It is very readable, using an engaging narrative style and a healthy sprinkling of biographical and historical background throughout. As the authors promise, the exercises do indeed have an "element of fun," and it is a rewarding book to work through.

A final word: This book is, to say the least, not typical of those generally reviewed on these pages. No doubt many readers of SIGACT news will ask "why should I (or any of my students) read it"? Here are some possible answers. One is that some of you may be employed in math departments where you will on occasion have to teach a course such as this. In that case, consider this text for adoption. Others, whose background is more purely in the computer science or mathematics camp, might simply be curious to learn more about mathematical physics. Consider that this book is primarily written from a mathematical point of view. Therefore, if your physics education is non-existent or has serious holes in it, you may very well want to start here.

[2] E.g., in his book *Gravitation and Cosmology* (pg. 85), Weinberg remarks that the calculation works if one ascribes a "gravitational potential energy" to a photon, but also notes that without an emitting particle such a concept is "without foundation."