



Test #1
Instructor: _____

Name: (print neatly) _____
(sign) _____

This exam is CLOSED NOTES and CLOSED BOOK. There are NO CALCULATORS allowed. To get full credit you must show all work neatly in the space provided on the test paper.

[10 pts each]

1. Compute the following integrals:

$$\begin{aligned} \text{a.} \quad \int 4x^3 - 3x + \frac{1}{x^2} dx &= \int 4x^3 - 3x^1 + x^{-2} dx \\ &= 4\frac{x^3}{3} - 3\frac{x^2}{2} + \frac{x^{-1}}{-1} + C \\ &= x^3 - \frac{3x^2}{2} - \frac{1}{x} + C \end{aligned}$$

$$\begin{aligned} \text{b.} \quad \int \frac{x}{\sqrt{1+4x^2}} dx &= \frac{1}{8} \int u^{-1/2} du = \frac{1}{8} \cdot \frac{u^{1/2}}{1/2} + C = \frac{1}{4} \cdot \sqrt{1+4x^2} + C \\ u &= 1 + 4x^2 \\ du &= 8x dx \\ (1/8)du &= x dx \end{aligned}$$

$$\begin{aligned} \text{c.} \quad \int \sin(3t) \cos(3t) dt &= \frac{1}{3} \int u du = \frac{1}{3} \cdot \frac{u^2}{2} + C = \frac{1}{6} \cdot \sin^2(3t) + C \\ u &= \sin(3t) \\ du &= 3 \cos(3t) dt \\ (1/3)du &= \cos(3t) dt \end{aligned}$$

$$\text{d.} \quad \int_0^1 \frac{t^3 + t}{5t} dt = \frac{1}{5} \int_0^1 t^2 + 1 dt = \frac{1}{5} \left(\frac{t^3}{3} + t \right) \Big|_0^1 = \frac{1}{5} \left(\frac{4}{3} \right) - 0 = \frac{4}{15}$$

$$\begin{aligned} \text{e.} \quad \int_{\pi}^{2\pi} \sin(\pi/6) - \sin(x/6) dx &= 6 \int_{\pi/6}^{\pi/3} \sin(\pi/6) - \sin(u) du = 6 (x \sin(\pi/6) + \cos(u)) \Big|_{\pi/6}^{\pi/3} \\ u &= x/6 \\ du &= (1/6) dx \\ 6du &= dx \\ u = \pi/6 & \quad x = \pi \\ u = \pi/3 & \quad x = 2\pi \end{aligned}$$

$$= 6 (x \sin(\pi/6) + \cos(u)) \Big|_{\pi/6}^{\pi/3} = 6((\pi/3) \sin(\pi/6) + (1/2)) - 6((\pi/6) \sin(\pi/6) + \frac{\sqrt{3}}{2})$$

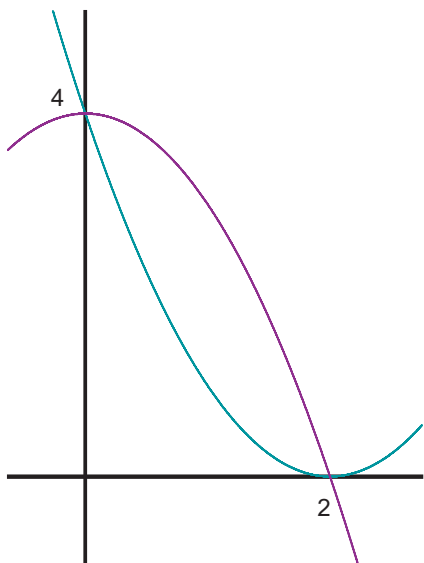
[10 pts]

2. Use the substitution $u = 1 + 3x$ to transform the integral $\int_0^2 x^2 \sin(x) \sqrt{3x} dx$ into an integral with respect to u . [Do not evaluate the integral.]

$$\begin{aligned}\int_0^2 x^2 \sin(x) \sqrt{3x} dx &= (1/3) \int_1^7 \left(\frac{u-1}{3}\right)^2 \sin\left(\frac{u-1}{3}\right) \sqrt{u-1} du \\ u &= 1 + 3x \\ du &= 3 dx \\ (1/3)du &= dx \\ \frac{u-1}{3} &= x \\ u = 1 & \quad x = 0 \\ u = 7 & \quad x = 2\end{aligned}$$

[12 pts]

3. Find the area of the finite region bounded by the graphs of the two functions $f(x) = x^2 - 4x + 4$ and $g(x) = 4 - x^2$.



Element Dimensions:

Width: Δx

Height: $(4 - x^2) - (x^2 - 4x + 4) = 4x - 2x^2$

Area: $(4x - 2x^2)\Delta x$

Total: $\int_0^2 (4x - 2x^2) dx = (2x^2 - 2x^3/3) \Big|_0^2 = (8 - 16/3) - (0) = 8/3$

[8 pts]

5. Suppose that $f'(x) = 1 - 3 \sin(3x)$ and $f(0) = 5$. Find $f(\pi/6)$.

$$f(x) = \int f'(x) dx = \int 1 - 3 \sin(3x) dx = x + \cos(3x) + C$$

Since $f(0) = 5$,

$$5 = f(0) = 0 + \cos(0) + C = 1 + C, \text{ so } C = 4.$$

So $f(x) = x + \cos(3x) + 4$, and

$$f(\pi/6) = \pi/6 + \cos(\pi/2) + 4 = \pi/6 + 4$$