

CLARK
UNIVERSITY



Name: _____
Circle your instructor's name:

Hill Joyce Pendharkar

Math 120 Calculus I
Final Exam
December 2015

This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2 \sqrt{\frac{\sin^2(\pi/6)}{1 + \ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [14; 7 points each part] Evaluate the following limits if they exist, but if a limit doesn't then explain why.

a. $\lim_{x \rightarrow \infty} \frac{e^x}{x^2 + 3x - 2}$

b. $\lim_{x \rightarrow 0^+} x \ln x$

2. [12; 4 points each part] Let $f(x) = \frac{x}{1+x^2}$.

a. Determine where $f'(x) = 0$.

b. Determine the intervals on which f is increasing and the intervals on which f is decreasing.

c. For which values of x are there local maxima, and for which values of x are there local minima.

3. [21; 7 points each part] Differentiation. Do not simplify your answers. Use parentheses properly.

a. Evaluate $\frac{d}{dx} \ln(1 + \sqrt{3x})$.

b. Let $f(t) = t^2 \sec t$. Find $f'(t)$.

c. Let $F(x) = \int_{-1}^x e^{t^2/2} dt$. Find $F'(x)$. (Hint: do not try to evaluate the integral.)

4. [8] Determine the function $f(x)$ that has the same derivative as $g(x) = 4e^x \cos x$ but whose value at $x = 0$ is $f(0) = 9$.

5. [8] Suppose that $f(x)$ is a differentiable function, and that $f(3) = 6$ and $f(8) = 5$. Explain why the derivative cannot always be positive. Mention appropriate theorem(s) to back up your explanation.

6. [14; 7 points each part] The second derivative of a function f is $f''(x) = \frac{x^2 - 1}{x^2 + 1}$
- a. For what values of x does f have an inflection point?

b. Determine the intervals on which f is concave upward, and the intervals on which it is concave downward.

7. [14; 7 points each part] Evaluate the following integrals. Note that the first one is an indefinite integral, and the second one is a definite integral. Show your work for credit.

a. $\int (4\sqrt{x} + \cos x) dx$

b. $\int_1^4 (x^3 - e^x) dx$

8. [12] The altitude of a triangle is increasing at a rate of 2 cm/min while the area of the triangle is increasing at a rate of 1 cm²/min. At what rate is the base of the triangle changing when the altitude is 5 cm and the area is 25 cm²? (Be sure to show your work.)

#1.[14]	
#2.[12]	
#3.[21]	
#4.[8]	
#5.[8]	
#6.[14]	
#7.[14]	
#8.[12]	
Total	