CLARK UNIVERSITY

Name: _____

Circle your instructor's name:

Hill Joyce Pendharkar

Math 120 Calculus I

First Test

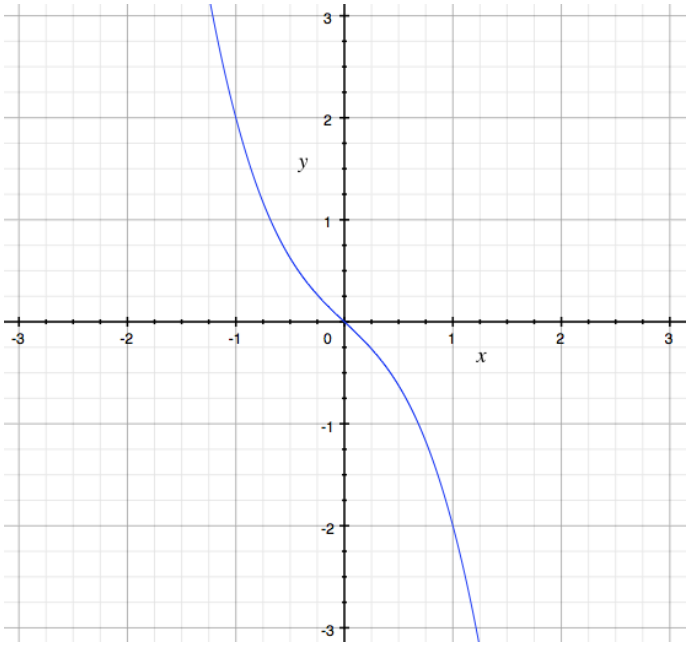
September 2015

This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [8] Determine the exact value of $\arctan\sqrt{3}$. Express your answer as a multiple of π radians. Explain in a sentence or a diagram why your answer is correct.

2. [8] The graph of $y = f(x) = -x^3 - x$ is shown below. On the same graph, sketch the graph of $y = f^{-1}(x)$.



3. [8] Note that both $\lim_{x\to 0} x = 0$ and $\lim_{x\to 0}(-x) = 0$. Determine the limit $\lim_{x\to 0} x\sin(1/x)$ and explain how you made your determination.

4. [32; 8 points each part] Evaluate the following limits. If a limit doesn't exist, then state briefly why not. (Use the properties of limits we've discussed so far in class. Later on this semester we'll discuss L'Hôpital's rule, but don't use that to determine these limits.)

a. $\lim_{x \to 2} \dfrac{x^2 + 4}{x^2 - 4}$

b. $\lim_{x \to 2} \dfrac{x^2 - 4}{x^2 + 3x + 10}$

c. $\lim_{x \to 0} \dfrac{3x}{\sin(4x)}$

d. $\lim_{x \to 0} \dfrac{\dfrac{1}{2+x} - \dfrac{1}{2}}{x}$

5. [14; 7 points each part] Consider the function $f(x) = 2x^3$.

a. Find the average rate of change of the f over the interval between 1 and b. Simplify your answer.

b. Use the information you found in part **a** to determine the instantaneous rate of change at $x = 1$.

6. [8] On one-sided limits. Draw the graph of a function for which $\lim_{x \to 1^-} f(x) = 2$ but $\lim_{x \to 1^+} f(x) = 4$.

7. [14; 7 points each part] Consider the function $f(x) = 5x - 2$. In this problem, you'll use the definition to prove that the limit $\lim_{x \to 2} f(x)$ is equal to 8.

a. Let $\epsilon = 0.1$. Find a positive value of δ so that if x is within δ of 1, then $f(x)$ must be within $\epsilon = 0.1$ of 8. Show your work or explain in words why your value of δ will work.

b. Now show it for arbitrary positive ϵ. This is like part **a**, but all you know about ϵ is that it's positive. Your value of δ will depend on ϵ.

8. [8] On continuity. Consider the function

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 5 \\ 10x & \text{if } x < 5 \end{cases}$$

f is not continuous at $x = 5$. Explain in terms of limits why f is not continuous there.

#1.[8]	
#2.[8]	
#3.[8]	
#4.[32]	
#5.[14]	
#6.[8]	
#7.[14]	
#8.[8]	
Total	