Name:
Circle your instructor's name:
Hill Joyce Pendharkar

## Math 120 Calculus I <br> Second Test

October 2015
This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^{2} \sqrt{\frac{\sin ^{2}(\pi / 6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [12] On implicit differentiation. The point $(1,1)$ lies in the curve $x^{2}-y^{2}=x y^{3}-1$. Determine the slope of the line tangent to the curve at that point.

2. [12] Recall the definition of derivatives in terms of limits, $f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$. Use that definition to show that the derivative of $f(x)=\frac{1}{x+1}$ is $f^{\prime}(x)=\frac{-1}{(x+1)^{2}}$. (Do not use any of the rules of differentiation, just the definition.)
3. [12] On logarithmic differentiation. The function $y=f(x)=x^{\sin x}$ cannot be differentiated by the power rule since the exponent is not constant, and it can't be differentiated by the exponential rule since the base is not constant, but you can find its derivative with logarithmic differentiation or by using the identity $a^{b}=e^{b \log a}$. Find its derivative. Show your work, and write carefully. Express your answer $f^{\prime}(x)$ in terms of $x$.
4. [40; 8 points each part] Differentiate the following functions. Do not simplify your answers. Use parentheses properly.
a. $f(t)=3 t^{5}-\frac{4}{t}+6+9 t^{2 / 3}$
b. $g(x)=e^{5 x}+\cos 3 x$
c. $y=\frac{3+4 \sqrt{x}}{5-\tan x}$
d. $f(x)=x \arcsin x$. (Note that the inverse sine function $\arcsin x$ is often written $\sin ^{-1} x$, but it does not equal $(\sin x)^{-1}$.)
e. $f(\theta)=\theta^{2} \sin 3 \theta$
5. [12] Consider the function $f(x)=\frac{x^{2}+x+1}{8 x^{2}-32}$.
a. Determine the limit $\lim _{x \rightarrow \infty} f(x)$.
b. Does $f(x)$ have any horizontal asymptotes? If yes, write the equation(s) for the horizontal line(s) that the curve $y=f(x)$ is asymptotic to.
c. Does $f(x)$ have any vertical asymptotes? If yes, write the equation(s) for the vertical line(s) that the curve $y=f(x)$ is asymptotic to.
6. [12] The power $P$ (watts) of an electric circuit is related to the circuit's resistance $R$ (ohms) and current $I$ (amperes) by the equation $P=R I^{2}$.
a. Assume that $R$ is constantly 20 ohms. Find an equation that relates $\frac{d P}{d t}$ to $\frac{d I}{d t}$.
b. If $R$ is constantly 20 ohms, $P$ is 5 watts, $\frac{d P}{d t}$ is 0.04 watts/sec, determine $\frac{d I}{d t}$ (which will be in units of amperes $/ \mathrm{sec}$ ).

| $\# 1 .[12]$ |  |
| :--- | :--- |
| $\# 2 .[12]$ |  |
| $\# 3 .[12]$ |  |
| $\# 4 .[40]$ |  |
| $\# 5 .[12]$ |  |
| $\# 6 .[12]$ |  |
| Total |  |

