CLARK UNIVERSITY

Name: _____

Circle your instructor's name:

Joyce Pendharkar Taylor Winders

Math 120 Calculus I

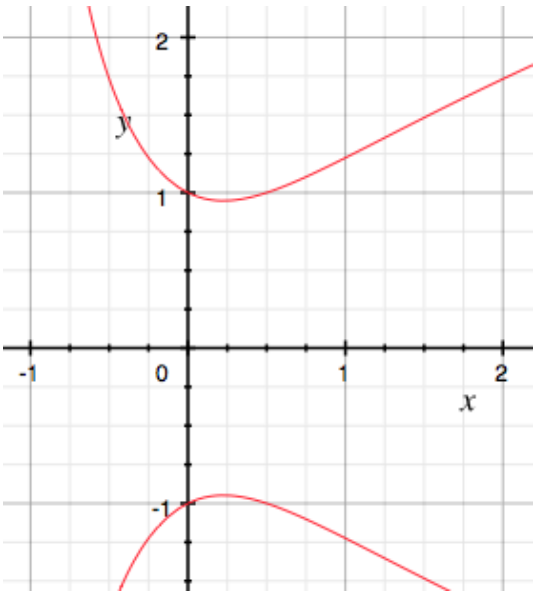
Second Test

November 2016

This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

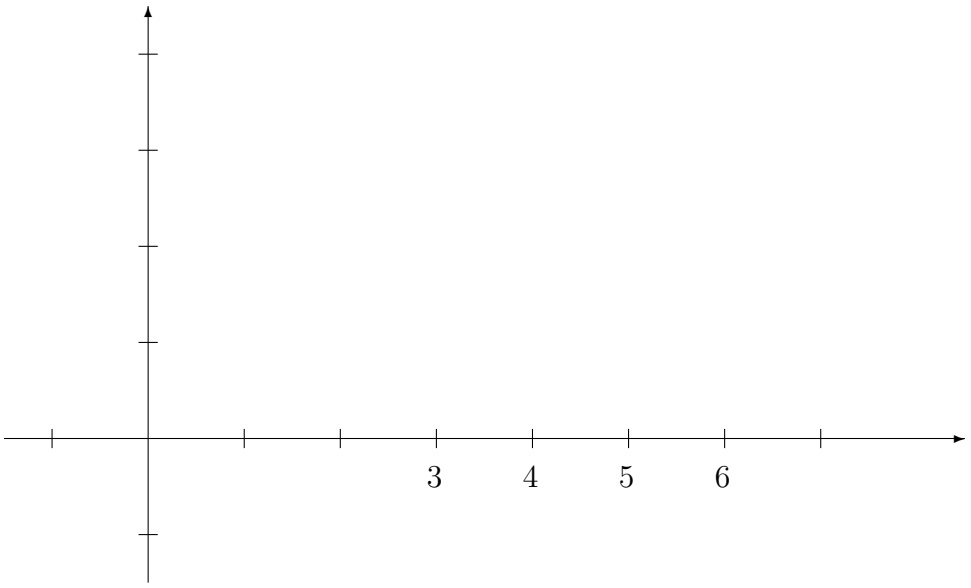
Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [10] On implicit differentiation. The point $(0, 1)$ lies in the curve $2x^2 = y^2(x + 1) - 1$. Determine the slope of the line tangent to the curve at that point.

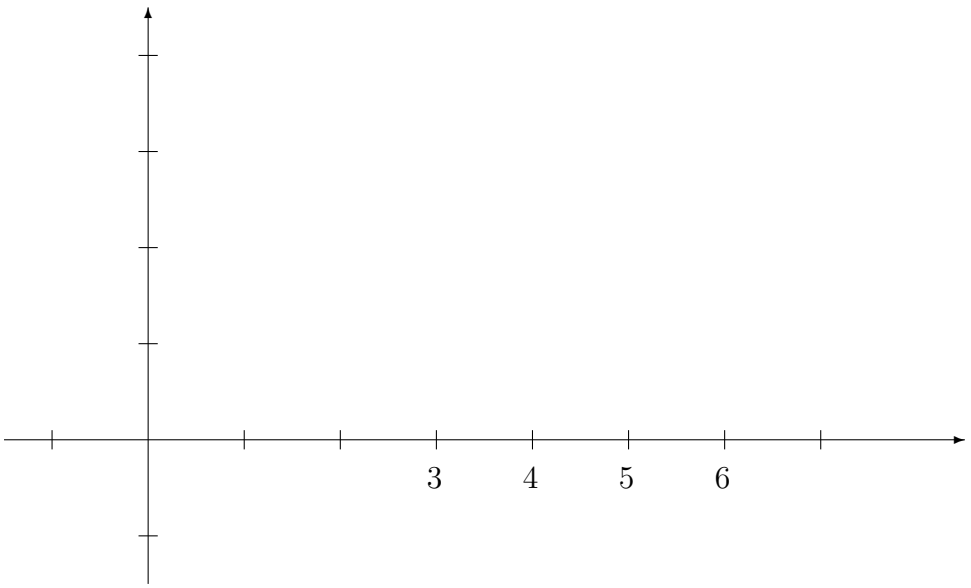


2. [10; 5 points each part] Graphs and derivatives.

a. Sketch the graph $y = f(x)$ of a function for which is continuous everywhere and differentiable everywhere except at $x = 4$.



b. Sketch the graph $y = f(x)$ of a function that is differentiable everywhere and whose derivative is 0 at $x = 3$ and $x = 5$ but the derivative is nonzero everywhere else.



3. [15] Recall the definition of derivatives in terms of limits, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$. Use that definition to show that the derivative of $f(x) = 5 + \dfrac{1}{x}$ is $f'(x) = -\dfrac{1}{x^2}$. (Do not use any of the rules of differentiation, just the definition.)

4. [12] On logarithmic differentiation. The function $y = f(x) = x^x$ cannot be differentiated by the power rule since the exponent is not constant, and it can't be differentiated by the exponential rule since the base is not constant, but you can find its derivative with logarithmic differentiation. Find its derivative. Show your work, and write carefully. Express your answer $f'(x)$ in terms of x.

5. [45; 9 points each part] Differentiate the following functions. Do not simplify your answers. Use parentheses properly.

a. $f(t) = 7t^4 - 4t + 6$

b. $g(x) = (3x^2 - 5)(\sin x)$

c. $y = \dfrac{e^x}{1 + x^2}$

d. $f(x) = x \arctan x$. (Note that the inverse tangent function $\arctan x$ is often written $\tan^{-1} x$, but it does not equal $(\tan x)^{-1}$.)

e. $f(x) = \sqrt{\ln x}$

6. [12] On related rates.

The formula for the volume of a circular cylinder is $V = \pi r^2 h$ where h is its height and r is the radius of the base.

Suppose that the height is constantly 10 inches, but the radius is increasing at the rate of 1 inch per minute. Determine the rate of change of the volume when the radius is 6 inches.

#1.[10]	
#2.[10]	
#3.[15]	
#4.[12]	
#5.[45]	
#6.[12]	
Total	