CLARK UNIVERSITY

Practice Differentiation
Math 120 Calculus I
Fall 2015

The rules of differentiation are straightforward, but knowing when to use them and in what order takes practice. Although the chain rule is no more complicated than the rest, it's easier to misunderstand it, and it takes care to determine whether the chain rule or the product rule is needed. For example, $\sqrt{x}\,\tan x$ and $\sqrt{x\tan x}$ look similar, but the first is a product while the second is a composition, so to differentiate the first, the product rule is needed but to differentiate the second, the chain rule is needed.

Here's a list of practice exercises. Differentiate each one using the various rules.

1. $\sqrt{x}\,\tan x$. Hint. Answer.

2. $\sqrt{x\tan x}$. Hint. Answer.

3. $\dfrac{5}{1-x}$. Hint. Answer.

4. $\dfrac{1-x}{5}$. Hint. Answer.

5. $\sin 2x + \cos 3x$. Hint. Answer.

6. $\sin(4\theta + \pi/2)$. Hint. Answer.

7. $\tan\theta + \sec\theta$. Hint. Answer.

8. $\sin 2x\,\cos 3x$. Hint. Answer.

9. $\sin(2\cos 3x)$. Hint. Answer.

10. $\sec^3 x^4$. Hint. Answer.

11. $\sqrt{1-x^2}$. Hint. Answer.

12. $(4x^2-9)\sqrt{4x^2+9}$. Hint. Answer.

13. $\dfrac{4x^2-9}{x^2-16}$. Hint. Answer.

14. $5x^{7/2} - 2x^{3/2} + 4x^{-3/2} - 8x^{-5/2}$. Hint. Answer.

15. $e^x \ln x$. Hint. Answer.

16. $e^{2x} + \ln(x^2+1)$. Hint. Answer.

17. $\sin e^t + \cos e^t$. Hint. Answer.

18. $\ln(\tan x + \sec x)$. Hint. Answer.

19. $\frac{1}{2}(e^t + e^{-t})$. Hint. Answer.

20. $\arctan(3x-5)$. Hint. Answer.

21. $\arcsin(3x-5)$. Hint. Answer.

22. $2^{x+\cos x}$. Hint. Answer.

1. Hint. $\sqrt{x}\tan x$. It's a product of the functions $\sqrt{x}$ and $\tan x$, so use the product rule,

$$(uv)' = u'v + uv'.$$

You can write the derivative of $\sqrt{x}$ either as $\dfrac{1}{2\sqrt{x}}$ or as $\frac{1}{2}x^{-1/2}$, whichever you prefer. Answer.

2. Hint. $\sqrt{x\tan x}$. This is a composition, so apply the chain rule first. The outer function is $\sqrt{}$, and the inner function is $x\tan x$. If you combine the chain rule with the derivative for the square root function, you get

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}.$$

In this exercise, when you compute the derivative of $x\tan x$, you'll need the product rule since that's a product. Answer.

3. Hint. $\dfrac{5}{1-x}$. It's a quotient, so you could use the quotient rule,

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}.$$

But the numerator is the constant 5, so the derivative is 5 times the derivative of $\dfrac{1}{1-x}$, and for that you could use a special case of the quotient rule called the reciprocal rule,

$$\left(\frac{1}{v}\right)' = \frac{-v'}{v^2}.$$

Answer.

4. Hint. $\dfrac{1-x}{5}$. This looks like a quotient, but since the denominator is a constant, you don't have to use the quotient rule. Rewrite it as $\frac{1}{5}(1-x)$ or as $\frac{1}{5}1 - \frac{1}{5}x$, and then it's easier to find its derivative. Answer.

5. Hint. $\sin 2x + \cos 3x$. You'll need the chain rule for each term. You may find it helpful to combine the basic rules for the derivatives of sine and cosine with the chain rule.

$$\frac{d}{dx}\sin u = (\cos u)\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -(\sin u)\frac{du}{dx}$$

Answer.

6. Hint. $\sin(4\theta + \pi/2)$. This is a composition, not a product, so use the chain rule. The outer function is sin, and the inner function is $4\theta + \pi/2$. Answer.

7. Hint. $\tan\theta + \sec\theta$. Remember that the derivative of $\tan\theta$ is $\sec^2\theta$, and the derivative of $\sec\theta$ is $\sec\theta\tan\theta$. Answer.

8. Hint. $\sin 2x\,\cos 3x$. This is the product of the two functions $\sin 2x$ and $\cos 3x$, so start by using the product rule. When you find the derivatives of $\sin 2x$ and of $\cos 3x$, be sure to use the chain rule. Answer.

9. Hint. $\sin(2\cos 3x)$. Although this may look like a product, it's not. It is not the product of sin and $2\cos 3x$ since sin is the name of a function. Instead, $\sin(2\cos 3x)$ is a composition where the outer function is sin and the inner function is $2\cos 3x$. You'll also need the chain rule for the derivative of $\cos 3x$. Answer.

10. Hint. $\sec^3 x^4$. This is an abbreviation for $(\sec(x^4))^3$, so it's a composition where the outer function is the cubing function, and the inner function is $\sec x^4$. Note that $\sec x^4$ is also a composition where the outer function is sec and the inner function is x^4, so you'll use the chain rule twice. Answer.

11. Hint. $\sqrt{1-x^2}$. Another composition where the outer function is $\sqrt{\quad}$ and the inner function is $1-x^2$. Answer.

12. Hint. $(4x^2-9)\sqrt{4x^2+9}$. You'll need both the product rule and the chain rule for this. The last operation that you would use to evaluate this expression is multiplication, the product of $4x^2-9$ and $\sqrt{4x^2+9}$, so begin with the product rule. Later on, you'll need the chain rule to compute the derivative of $\sqrt{4x^2+9}$. Answer.

13. Hint. $\dfrac{4x^2-9}{x^2-16}$. Apply the quotient rule. Answer.

14. Hint. $5x^{7/2}-2x^{3/2}+4x^{-3/2}-8x^{-5/2}$. The power rule

$$(x^n)' = nx^{n-1}$$

works for fractional powers n. Just remember that n has to be a constant, as it is here in each of the four terms. Answer.

15. Hint. $e^x \ln x$. Use the product rule and the basic rules for the exponential and logarithmic functions: $(e^x)' = e^x$ and $(\ln x)' = 1/x$. Answer.

16. Hint. $e^{2x} + \ln(x^2+1)$. You'll need the chain rule to evaluate the derivative of each term. You may find it helpful to combine the chain rule with the basic rules of the exponential and logarithmic functions:

$$(e^u)' = e^u u' \text{ and } (\ln u)' = \frac{u'}{u}.$$

Answer.

17. Hint. $\sin e^t + \cos e^t$. The terms are both compositions where the outer function is a trig function, and the inner function is e^t. Answer.

18. Hint. $\ln(\tan x + \sec x)$. This is a composition where the outer function is ln and the inner function is $\tan x + \sec x$. Note that it's *not* a product, ln times $\tan x + \sec x$, since ln is the name of a function and requires an argument. Answer.

19. Hint. $\frac{1}{2}(e^t + e^{-t})$. When computing the derivative of e^{-t}, don't forget to use the chain rule. Answer.

20. Hint. $\arctan(3x-5)$. The arctangent function is also denoted $\tan^{-1}$, but that notation suggests that $\tan^{-1} x$ should be the same as $1/\tan x$, which it isn't. The notation arctan is unambiguous. The expression $\arctan(3x-5)$ indicates a composition of two functions. It's not a product. So, you'll need the chain rule. Here's the chain rule combined with the rule for differentiating arctan:

$$(\arctan u)' = \frac{u'}{1+u^2}$$

Answer.

21. Hint. $\arcsin(3x-5)$. The arcsine function is also denoted $\sin^{-1}$. The chain rule combined with the rule for differentiating arcsin is

$$(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$$

Answer.

22. Hint. $2^{x+\cos x}$. There are at least three different ways of finding the derivative. One is apply the rule

$$(a^x)' = a^x \ln a$$

which applies whenever the base a is constant. That requires memorizing that rule.

A second method uses the identity

$$a^x = e^{x \ln a}$$

which is useful to convert exponentiation with different bases to exponentiation with the natural base e.

A third method is logarithmic differentiation which is useful whenever an expression involves exponentiation, products, quotients, powers, and roots. Although it works here, the first two methods are easier for this function. Answer.

These computations show how to compute the derivatives. Intermediate stages are shown to illustrate when each rule is used.

1. Answer. $\sqrt{x}\,\tan x$.

$$\begin{aligned}(\sqrt{x}\,\tan x)' &= (\sqrt{x})'\tan x + \sqrt{x}\,(\tan x)' \\ &= \frac{1}{2\sqrt{x}}\tan x + \sqrt{x}\,\sec^2 x\end{aligned}$$

2. Answer. $\sqrt{x\tan x}$.

$$\begin{aligned}(\sqrt{x\tan x})' &= \frac{1}{2\sqrt{x\tan x}}(x\tan x)' \\ &= \frac{1}{2\sqrt{x\tan x}}(\tan x + x\sec^2 x)\end{aligned}$$

3. Answer. $\dfrac{5}{1-x}$.

$$\left(\frac{5}{1-x}\right)' = \frac{5(1-x)'}{(1-x)^2} = \frac{-5}{(1-x)^2}$$

4. Answer. $\dfrac{1-x}{5}$.

$$(\tfrac{1}{5}1 - \tfrac{1}{5}x)' = -\tfrac{1}{5}$$

5. Answer. $\sin 2x + \cos 3x$.

$$(\sin 2x + \cos 3x)' = 2\cos 2x - 3\sin 3x$$

6. Answer. $\sin(4\theta + \pi/2)$.

$$\begin{aligned}\sin(4\theta+\pi/2) &= (\cos(4\theta+\pi/2))\,(4\theta+\pi/2)' \\ &= 4\cos(4\theta+\pi/2)\end{aligned}$$

7. Answer. $\tan\theta + \sec\theta$.

$$(\tan\theta + \sec\theta)' = \sec^2\theta + \sec\theta\tan\theta$$

8. Answer. $\sin 2x\,\cos 3x$.

$$\begin{aligned}&(\sin 2x\,\cos 3x)' \\ =\;& (\sin 2x)'(\cos 3x) + (\sin 2x)(\cos 3x)' \\ =\;& (\cos 2x)(2x)'(\cos 3x) + (\sin 2x)(-\sin 3x)(3x)' \\ =\;& 2\cos 2x\,\cos 3x - 3\sin 2x\,\sin 3x\end{aligned}$$

9. Answer. $\sin(2\cos 3x)$.

$$\begin{aligned}(\sin(2\cos 3x))' &= (\cos(2\cos 3x))\,(2\cos 3x)' \\ &= (\cos(2\cos 3x))\,2(-\sin 3x)(3x)' \\ &= -6\sin 3x\ \cos(2\cos 3x)\end{aligned}$$

10. Answer. $\sec^3 x^4$.

$$\begin{aligned}(\sec^3 x^4)' &= 3(\sec^2 x^4)\,(\sec x^4)' \\ &= 3(\sec^2 x^4)\,(\sec x^4)(\tan x^4)\,(x^4)' \\ &= 12x^3\sec^3 x^4\ \tan x^4\end{aligned}$$

11. Answer. $\sqrt{1-x^2}$.

$$\begin{aligned}\left(\sqrt{1-x^2}\right)' &= \frac{1}{2\sqrt{1-x^2}}\,(1-x^2)' \\ &= \frac{-2x}{\sqrt{1-x^2}}\end{aligned}$$

12. Answer. $(4x^2-9)\sqrt{4x^2+9}$.

$$\begin{aligned}&\left((4x^2-9)\sqrt{4x^2+9}\right)' \\ &= (4x^2-9)'\sqrt{4x^2+9}+(4x^2-9)\left(\sqrt{4x^2+9}\right)' \\ &= 8x\sqrt{4x^2+9}+(4x^2-9)\frac{1}{2\sqrt{4x^2+9}}(4x^2+9)' \\ &= 8x\sqrt{4x^2+9}+\frac{8x(4x^2-9)}{2\sqrt{4x^2+9}}\end{aligned}$$

If you like, you can simplify that further.

13. Answer. $\dfrac{4x^2-9}{x^2-16}$.

$$\begin{aligned}&\left(\frac{4x^2-9}{x^2-16}\right)' \\ &= \frac{(4x^2-9)'(x^2-16)-(4x^2-9)(x^2-16)'}{(x^2-16)^2} \\ &= \frac{8x(x^2-16)-(4x^2-9)2x}{(x^2-16)^2}\end{aligned}$$

which can be simplified.

14. Answer. $5x^{7/2}-2x^{3/2}+4x^{-3/2}-8x^{-5/2}$. The derivative is

$$\tfrac{35}{2}x^{5/2}-6x^{1/2}-6x^{-5/2}+20x^{-7/2}.$$

15. Answer. $e^x\ln x$.

$$(e^x\ln x)' = (e^x)'\ln x+e^x(\ln x)' = e^x\ln x+e^x/x$$

16. Answer. $e^{2x}+\ln(x^2+1)$.

$$\begin{aligned}\left(e^{2x}+\ln(x^2+1)\right)' &= e^{2x}(2x)'+\frac{(x^2+1)'}{x^2+1} \\ &= 2e^{2x}+\frac{2x}{x^2+1}\end{aligned}$$

17. Answer. $\sin e^t+\cos e^t$.

$$\begin{aligned}(\sin e^t+\cos e^t)' &= (\cos e^t)(e^t)'-(\sin e^t)(e^t)' \\ &= e^t(\cos e^t-\sin e^t)\end{aligned}$$

18. Answer. $\ln(\tan x+\sec x)$.

$$\begin{aligned}(\ln(\tan x+\sec x))' &= \frac{1}{\tan x+\sec x}(\tan x+\sec x)' \\ &= \frac{\sec^2 x+\sec x\tan x}{\tan x+\sec x}\end{aligned}$$

Note that you can factor the numerators as $\sec x(\sec x+\tan x)$ so that you can simplify the answer to $\sec x$. Thus, the derivative of $\ln(\tan x+\sec x)$ is $\sec x$. That will become important later when we're looking for antiderivatives of trig functions.

19. Answer. $\frac{1}{2}(e^t+e^{-t})$. The derivative is

$$\frac{1}{2}(e^t-e^{-t}).$$

The function $\frac{1}{2}(e^t+e^{-t})$ is also called the *hyperbolic cosine*, denoted $\cosh t$, while the function $\frac{1}{2}(e^t+e^{-t})$ is also called the *hyperbolic sine*, denoted $\sinh t$. This exercise showed $(\cosh t)'=\sinh t$. Similarly, you can show $(\sinh t)'=\cosh t$. The hyperbolic functions are like the trigonometric functions in many ways. In fact, with the use of complex numbers you can show they're both aspects of of the same complex functions.

20. Answer. $\arctan(3x-5)$.

$$(\arctan(3x-5))' = \frac{(3x-5)'}{1+(3x-5)^2} = \frac{3}{1+(3x-5)^2}$$

which can be simplified if you like.

21. Answer. $\arcsin(3x-5)$.

$$(\arcsin(3x-5))' = \frac{(3x-5)'}{\sqrt{1-(3x-5)^2}} = \frac{3}{\sqrt{1-(3x-5)^2}}$$

which can be simplified if you like.

22. Answer. $2^{x+\cos x}$. Using the rule $(a^x)' = a^x \ln a$ combined with the chain rule, you find

$$\begin{aligned}(2^{x+\cos x})' &= 2^{x+\cos x}(\ln 2)(x+\cos x)' \\ &= 2^{x+\cos x}(\ln 2)(1-\sin x)\end{aligned}$$

Math 120 Home Page at
`http://math.clarku.edu/~ma120/`