Differentiation Rules
Math 120 Calculus I
Fall 2015

The great thing about the rules of differentiation is that the rules are complete. There are relatively few of them, as well. If a function is given to you as a formula, then you can find the derivative.

You'll use the rules for constants, addition, subtraction, and constant multiples automatically. The derivative of a constant function c is 0 because it doesn't change. The derivative of a sum or difference of two functions is the sum or difference of their derivatives, respectively. The derivative of a constant times a function is that constant times the derivative of the function.

$$\boxed{\begin{array}{cc} c' = 0 & (f+g)' = f' + g' \\ (cf)' = c\,f' & (f-g)' = f' - g' \end{array}}$$

The first interesting rule is the power rule. It works for any numerical power n.

$$\boxed{(x^n)' = nx^{n-1}}$$

Note that since $\sqrt{x}$ is the same as $x^{1/2}$, a special case of the power rule is

$$\boxed{\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}}$$

With this power rule and the basic rules mentioned above you can easily find the derivative of any polynomial. For example $(7x^5 - 4x^3 + 5x - 6)' = 35x^4 - 12x^2 + 5$. More generally, you can use these rules to find the derivative of a sum of multiples of any powers of x. So, for example $\left(14x^{10/7} + \frac{5}{x} + \frac{8}{\sqrt{x}}\right)' = 20x^{3/7} - \frac{5}{x^2} - \frac{4}{x\sqrt{x}}$.

Many functions are built out of other functions using products and quotients, and the product and quotient rules allow you to find their derivatives. A special case of the quotient rule is the reciprocal rule. It comes up often enough to treat it as a separate rule.

$$\boxed{(fg)' = f'g + fg' \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \qquad \left(\frac{1}{g}\right)' = \frac{-g'}{g^2}}$$

Finally, functions are constructed from other functions using composition. The chain rule says how to find the derivative of a composition of functions.

$$\boxed{(f \circ g)' = f'(g)\,g'}$$

Transcendental functions aren't built out of simpler functions, so each requires its own rule. The important transcendental functions include the trig functions and inverse trig functions, logarithms, and exponential functions. The derivatives of the important trig functions and inverse trig functions are

$$
\boxed{\begin{array}{ll}
(\sin x)' = \cos x & (\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}} \\
(\cos x)' = -\sin x & \\
(\tan x)' = \sec^2 x & (\arctan x)' = \dfrac{1}{1+x^2} \\
(\sec x)' = \sec x\ \tan x &
\end{array}}
$$

The derivatives of the exponential and logarithmic functions are

$$
\boxed{\begin{array}{ll}
(e^x)' = e^x & (a^x)' = a^x \ln a \\
(\ln x)' = \dfrac{1}{x} & (\log_a x)' = \dfrac{1}{x \ln a}
\end{array}}
$$

We won't prove these last four rules are correct until we discuss integrals where they'll be easy to prove.

With these few rules you can differentiate almost any function we'll come across this semester.

Math 120 Home Page at `http://math.clarku.edu/~ma120/`