Name: ____________________________

Circle your instructor's name:

Broker Hill Joyce Pendharkar

Math 121 Calculus II

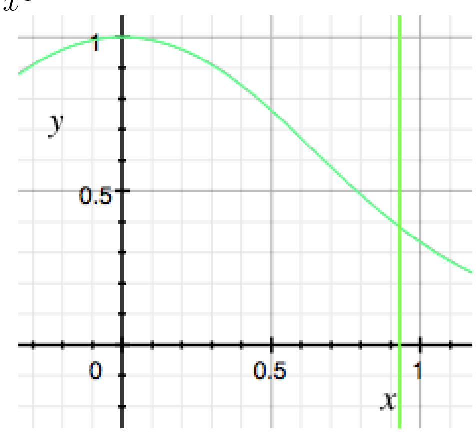
First Test

February 2015

This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [10] **On integrals and derivatives.** For a positive number x, consider the region under the curve $y = \dfrac{1}{1+x^2+x^4}$, above the x-axis, to the right of the y-axis, and to the left of the vertical line at x. Denote the area of that region as $F(x)$. Determine the derivative $F'(1)$. (Note that this isn't asking for the derivative of $\dfrac{1}{1+x^2+x^4}$ at $x=1$.)



2. [20; 10 points each part] **Evaluate the indefinite integrals.**

a. $\displaystyle\int \left(8x^3 - 6x^2 - 3\sqrt{x} + \frac{2}{x^3} + \frac{1}{x}\right) dx$

b. $\displaystyle\int xe^{x^2+5}\, dx$

3. [20; 10 points each part] **On definite integrals.** Evaluate the following integrals. Show your work for credit. (You do not have to find the answer decimally; an unsimplified expression involving numbers is sufficient.)

a. $\displaystyle\int_1^2 \frac{2\ln x}{x}\,dx$

b. $\displaystyle\int_1^4 \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx$

4. [10; 5 points each part] **On areas between curves.** Consider the region in the plane between the parabolas $y = 4x^2$ and $y = x^2 + 3$.

a. Roughly sketch these two parabolas and determine their points of intersection.

b. Write down an integral which gives the area of that region. Do not compute the value of that integral.

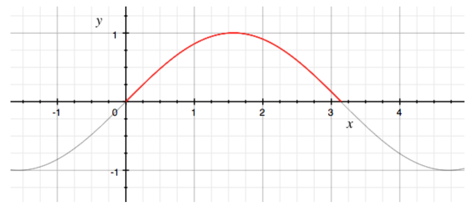
5. [20] **On volumes of solids of revolution.** In each case write down an expression for the volume of this solid in terms of integrals. Don't valuate the integrals.

a. The solid of revolution is formed by rotating about the y-axis the region bounded by the y-axis and the curve $x = y^2 - 4$.

b. The solid of revolution is formed by rotating around the x-axis the region between the two curves $y = x$ and $y = (x-2)^2$.

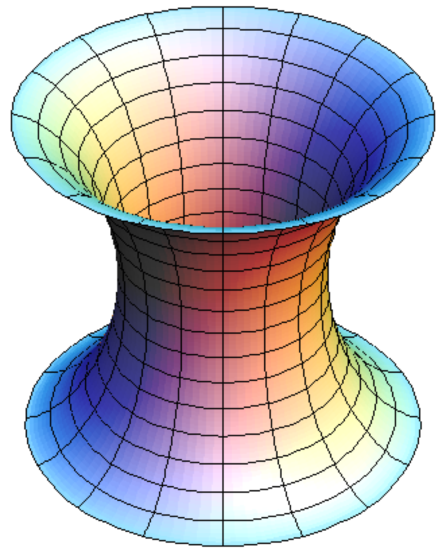
6. [10] **On arc lengths.** Recall that if a curve is given by the equation $y = f(x)$ for $a \leq x \leq b$, then the length of the curve can be described by the integral $\int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\, dx$.

Express the length of one bump of the curve $y = \sin x$ as an integral. Do not evaluate the integral.



7. [10] **On areas of surfaces of revolution.** Recall that if a curve given by the equation $y = f(x)$ for $a \le x \le b$ is rotated around the x-axis to generate a surface of revolution, then the area of that surface can be described by the integral $\int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx$.

Suppose that the curve $x = 1 + y^2$ for $-1 \le y \le 1$ is rotated around the y-axis to generate a surface of revolution. Express the area of that surface in terms of an integral. Do not evaluate the integral.



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