Name: ______________________________

Circle your instructor's name:

Hill Joyce Winders

Math 121 Calculus II

Final Exam

May 2016

This is a closed-book, closed-notes test. There are some useful formulas and theorems on the last page of the test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [6] Determine the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{3n}\, x^n$. (For this problem it is not necessary to determine whether the series converges at the endpoints of the interval of convergence.)

2. [12; 6 points each part] **On arc length.** Consider the curve $y = \ln(\cos x)$ for $0 \le x \le \pi/4$.

a. Write down an integral which gives the length of that curve.

b. Evaluate that integral.

3. [21; 7 points each part] **On integration.** Evaluate the integrals. Note that the first and third are a indefinite integrals while the second is a definite integral.

a. $\displaystyle\int x\cos x\,dx.$

b. $\displaystyle\int_0^9 \frac{dx}{\sqrt{1+\sqrt{x}}}.$ [Hint: substitution]

c. $\displaystyle\int \frac{7x-3}{(x-2)(x+1)}\,dx$

4. [10; 5 points each part] **On sequences.** State whether each sequence converges or diverges and explain why. The general term a_n is given for the sequence. If the sequence converges, find it's limit. If you use l'Hôpital's rule, point out where you use it.

a. $a_n = \dfrac{3n^3 - 4n^2 + 6}{2n^3 + 5n^2 + 10}$

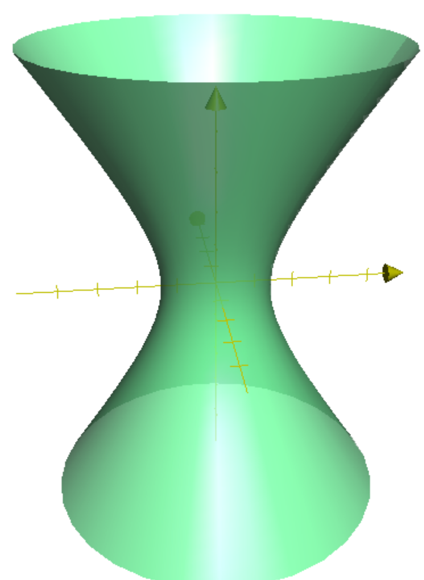
b. $a_n = \dfrac{\ln n^3}{n}$

5. [10; 5 points each part] **On series.**

a. Give an example of a (nontrivial) geometric series with ratio $r = 2/3$, and give its sum.

b. Give an example of a geometric series that diverges. (You don't have to prove it diverges; just give an example.)

6. [8] **On volumes.** Consider the region in the xy-plane bounded on the left by the y-axis, on the right by the curve $x = \sqrt{1+y^2}$, above by the line $y = 3$, and below by the line $y = -3$. Rotate this about the y-axis to get the solid shown. Write down an integral which gives the volume of this solid. Do not evaluate that integral.



7. [12; 6 points each part] **On convergence of positive series.** For each series, apply one or more convergence tests to determine whether the series converges. State which test(s) you use and explain your conclusion for credit.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 + \ln n}$

b. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$. [Hint: integral test]

8. [12; 4 points each part] **On alternating series, conditional convergence, and absolute convergence.** Give an example for each of the following. You don't have to prove that you're correct; the example is sufficient by itself. You may use summation notation or write out several terms with ellipsis $\cdots$ notation.

a. An alternating series which diverges.

b. An alternating series which conditionally converges.

c. An alternating series which absolutely converges.

9. [6] **On improper integrals** Determine whether the following improper integral converges or diverges, and evaluate it if it does converge.

$$\int_0^1 \frac{dx}{x^2}$$

10. [6] Determine the Taylor polynomial of order 3 (the terms of the Taylor series up through x^3) generated by the function $f(x) = \sin x + \cos x$ centered at $x = 0$.

#1.[6]	
#2.[12]	
#3.[21]	
#4.[10]	
#5.[10]	
#6.[8]	
#7.[12]	
#8.[12]	
#9.[6]	
#10.[6]	
Total	