Name: ______________________________

Math 121 Calculus II

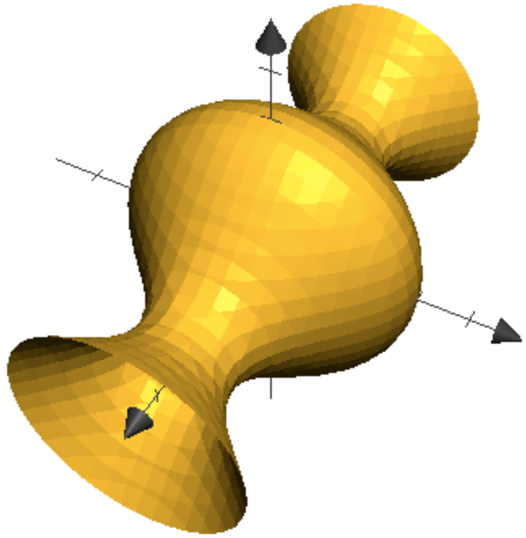
Second Test

March 2016

This is a closed-book, closed-notes test. There are some useful formulas on the last page of the test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [13] **On areas of surfaces of revolution.** The curve $y = 2+\cos x$, for x in the interval $[-3,3]$, is rotated around the x-axis to generate a surface of revolution. Write down an integral which gives the area of this wavy surface. Do not evaluate the integral.



2. [15] **On exponential functions.** A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. Initially there were 1000 bacteria, and after 4 hours there are 4000 bacteria. How many bacteria were there after 3 hours?

3. [15] **On differential equations.** Use the method of separation of variables to solve the differential equation

$$\frac{dy}{dx} = x(1 + y^2).$$

Your answer should be in the form $y = f(x)$ where the function f involves a constant C of integration.

4. [30] **On integration.** Evaluate the following integrals. Show your work for credit.

a. $\displaystyle\int \theta \cos\theta \, d\theta$

b. $\int \arctan x \, dx$

c. $\int \frac{\sqrt{x^2 - 16}}{x^4} \, dx$

5. [18] **On partial fractions.**

a. The rational function $\dfrac{5x-4}{(x+1)(x-2)}$ is equal to the sum of the partial fractions $\dfrac{A}{x+1}+\dfrac{B}{x-2}$. Determine the values of A, B.

b. Use your answer in part a to evaluate the integral

$$\int \frac{5x-4}{(x+1)(x-2)}\,dx$$

6. [15] **On Improper integrals** Use the fact that $\int xe^{-x}\,dx = -(x+1)e^{-x}+C$ to evaluate the improper integral $\int_1^{\infty} xe^{-x}\,dx$. Explain in a sentence how you used limits to find your answer.

#1.[13]	
#2.[15]	
#3.[13]	
#4.[30]	
#5.[18]	
#6.[15]	
Total	

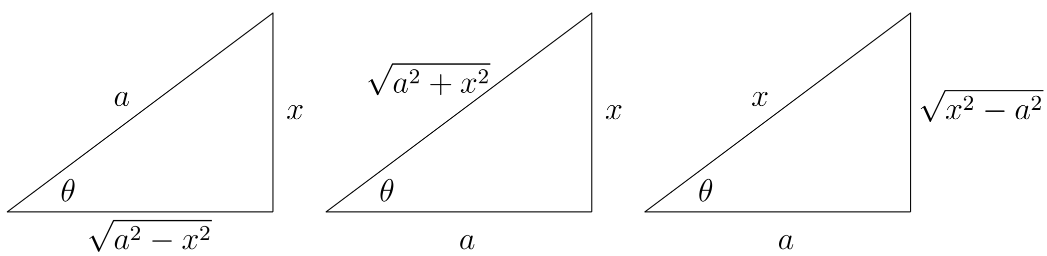
Some useful formulas

Trig identities

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Trig subs

$x = a\sin\theta$	$x = a\tan\theta$	$x = a\sec\theta$
$dx = a\cos\theta\, d\theta$	$dx = a\sec^2\theta\, d\theta$	$dx = a\sec\theta\tan\theta\, d\theta$
$\sqrt{a^2-x^2} = a\lvert\cos\theta\rvert$	$\sqrt{a^2+x^2} = a\lvert\sec\theta\rvert$	$\sqrt{x^2-a^2} = a\lvert\tan\theta\rvert$



Some useful integrals Most of those on page 431 of the text you should know (especially 1–7). Here are some others

$$\int \sec^2 x\, dx = \tan x + C$$
$$\int \csc^2 x\, dx = -\cot x + C$$
$$\int \sec x \tan x\, dx = \sec x + C$$
$$\int \csc x \cot x\, dx = -\csc x + C$$
$$\int \tan x\, dx = \ln|\sec x| + C$$
$$\int \cot x\, dx = \ln|\sin x| + C$$
$$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$
$$\int \csc x\, dx = -\ln|\csc x + \cot x| + C$$