CLARK UNIVERSITY 1887

Integration by Parts
Math 121 Calculus II
Spring 2015

This is just a short note on the method used in integration called integration by parts. It corresponds to the product rule for differentiation.

Let's start with the product rule and convert it so that it says something about integration. We'll use Leibniz' notation. If u and v are functions of x, the product rule says

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx},$$

in other words, the right hand side is an antiderivative of uv

$$uv = \int \left(u\frac{dv}{dx} + v\frac{du}{dx} \right) dx.$$

Using the sum rule and the substitution rule for integrals, we can rewrite that as

$$uv = \int u\,dv + \int v\,du.$$

To make this into a useful rule for integration, we'll write it as

$$\boxed{\int u\,dv = uv - \int v\,du}$$

and call it the method of integration by parts, abbreviated simply as "by parts."

An example. Consider the integral $\int x\cos x\,dx$. Let $u = x$, and let $dv = \cos x\,dx$. Then $du = dx$, and $v = \sin x$. We have, by parts,

$$\int x\cos x = x\sin x - \int \sin x\,dx.$$

That last integral is easy to integrate, and we have the answer, $x\sin x + \cos x + C$.

Note that our original integrand, $x\cos x$ was a product, and we integrated one term of that product, namely, $\cos x$, when we applied the method of integration by parts. That's why it's called that, you integrate one part of the integrand. With any luck, you've simplified the problem.

Be careful, though, you could complicate the problem if you use the wrong part. Suppose we had chosen to let $u = \cos x$ and $dv = x\,dx$. Then $du = -\sin x\,dx$, and $v = \frac{1}{2}x^2$. By parts, we would then have

$$\int x\cos x = -\tfrac{1}{2}x^2\sin x + \int \tfrac{1}{2}x^2\sin x\,dx,$$

and the new integral is harder to solve then the original one was.

Generally speaking, why you apply parts, you want to choose u so that is simplifies when you differentiate it, and choose dv so that, at least, you can integrate it.

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