Useful formulas

Some indefinite integrals

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\left|\frac{x}{a}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln(x + \sqrt{a^2 + x^2}) + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$$

Some limits

$$\lim_{n\to\infty} \frac{\ln n}{n} = 0$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} x^{1/n} = 1 \text{ if } x > 0$$

$$\lim_{n\to\infty} x^n = 0 \text{ if } |x| < 1$$

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$$

$$\lim_{n\to\infty} \frac{x^n}{n!} = 0$$

The average value of a function f on an interval $[a, b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Volume of a solid in terms of area of the cross section at x

$$\int_a^b A(x)\,dx$$

Volumes of solids of revolution about the x-axis

1. Disk method. $\displaystyle\int_a^b \pi(R(x))^2\,dx$
2. Washer method. $\displaystyle\int_a^b \pi\left((R(x))^2 - (r(x))^2\right)dx$

Length of a curve $y = f(x)$ for $a \le x \le b$

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Area of surface of revolution about the x-axis

$$\int_a^b 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Convergence tests for series

Integral test. If f is a positive decreasing function, and $a_n = f(n)$, then the series $\displaystyle\sum_{n=N}^{\infty} a_n$ and the integral $\displaystyle\int_N^{\infty} f(x)\,dx$ either both converge or both diverge.

p-series. The series $\sum 1/n^p$ converges if $p > 1$ but diverges if $p \leq 1$.

Comparison test. Given two series of nonnegative terms, $\sum a_n$ and $\sum b_n$ where $a_n \leq b_n$ for all $n > N$,

1. if $\sum b_n$ converges, then so does $\sum a_n$.
2. if $\sum a_n$ diverges, then so does $\sum b_n$.

Limit comparison test. Given two series of nonnegative terms, $\sum a_n$ and $\sum b_n$

1. if $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then $\sum a_n$ and $\sum b_n$ either both converge or both diverge
2. if $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then so does $\sum a_n$.
3. if $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then so does $\sum a_n$.

Ratio test. Given a series $\sum a_n$, if $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = r$, then

1. if $r < 1$, then the series absolutely converges.
2. if $r > 1$, then the series diverges.
3. if $r = 1$, then the test is inconclusive so try another test.

Root test. Given a series $\sum a_n$, if $\displaystyle\lim_{n\to\infty} \sqrt[n]{|a_n|} = r$, then

1. if $r < 1$, then the series absolutely converges.
2. if $r > 1$, then the series diverges.
3. if $r = 1$, then the test is inconclusive so try another test.

Alternating series test. An alternating series $a_1 - a_2 + \cdots + (-1)^{n+1} a_n + \cdots$ converges if each a_n is positive, $a_{n+1} \leq a_n$ for $n > N$, and $\lim_{n\to\infty} a_n = 0$.

Absolute convergence test. If $\sum |a_n|$ converges, then so does $\sum a_n$.

Taylor's formula for the n^{th} coefficient a_n of a Taylor polynomial/series $\sum a_n (x-a)^n$ is

$$a_n = \frac{f^{(n)}(a)}{n!}$$